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THEORETICAL CALCULATION OF LOADED SHELLS AND COMPARISON WITH OBTAINED NUMERICAL RESULTS

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ABSTRACT
In this paper is given bending theory for cylindrical shell subjected to uniformly distributed couple along helix. At some machines for special purposes particularly at machines for snow cleansing, can be found structural elements of thin cylindrical shells on which are welded helicoidally shells. We assume that the displacement due to force are small compared to those due to torque and the helix are simply supported. For curvilinear coordinates on cylindrical surface are introduced angles φ, ψ, so that the coordinate lines are two families or orthogonal helices. Unknown coefficient in the solution of homogeneous linear differential equation with constant coefficients are determined on basis of four boundary conditions. The location of points on cylindrical shell with maximum displacement is found as well as magnitude of this displacements, it is shown that this location is independent on couple. Previous results are compared with those obtained by FEA software code Pro/MECHANICA which has capability of defining uniformly distributed couple along helix. A good accordance between theoretical and numerical results is obtained..

Keywords: shell, helicoidall, plate, cilindrical, strains, force per unit length, couple per unit length

1. INTRODUCTION
With special machines, such as machines for snow cleansing, helicoidal transporeters and similar, structural elements of thin cylindrical shell on which helicidal shell are welded, can be found. Pressure on helicoidal shell is reduced to cylindrical one by cross-section line of medial surfaces, namely by the helix. In case when helicoidal surfaces is loaded by pressure, the resultant is such load on cylinder is of a continually distributed force in the tangent cylinder plane and a continual couple in tangent direction on helicoidal line on cylinder. The force impact on displacement is neglected in this paper, so we consider the bending theory of cylindrical shell loaded by a continual couploe along helix.

2. THEORETICAL CALCULATION OF LOADED SHELLS
2.1. Parameters of cylindrical surface within φ, ψ coordinate system
As the helix is a curve along which boundary conditions are given it is necessary to introduce curvilinear coordinates on cylindrical surfaces, these being different from the ones that are most natural for the cylinder, and they are the angle and distance along the leaher. As curvilinear

coordinates on medial shell surfaces two angles are introduced $\theta_1 \equiv \varphi$, $\theta_2 \equiv \psi$, so that coordinates are the lines of two helix fammilies. Parameter equation of medium cylindrical shell surface is [1]:

$$x = R\cos(\varphi+\psi), \qquad y = R\sin(\varphi+\psi), \qquad z = k\varphi - \frac{R^2}{k}\psi \tag{1}$$

Basic vectors are:

$$A_1 = \frac{\partial R}{\partial \varphi} = -iR\sin(\varphi+\psi) + jR\cos(\varphi+\psi) + kk, \quad A_2 = \frac{\partial R}{\partial \psi} = -iR\sin(\varphi+\psi) + jR\cos(\varphi+\psi) - \frac{R^2}{kk} \tag{2}$$

where R is a point position vector on cylinder. A unit vertical vector is:

$$A_3 = \frac{A_1 \times A_2}{|A_1 \times A_2|} = -i\cos(\varphi+\psi) - j\sin(\varphi+\psi) \tag{3}$$

C_0-variant coefficients of the first fundamental surfaces form are:

$$A_{11} = A_1 \cdot A_1 = R^2 + k^2, \quad A_{12} = A_{21} = A_1 \cdot A_2 = 0, \quad A_{22} = A_2 \cdot A_2 = \frac{R^2}{k^2}\left(R^2 + k^2\right) \tag{4}$$

whereas contra-variant coefficient are:

$$A^{11} = \frac{1}{R^2 + k^2}, \quad A^{12} = A^{21} = 0, \quad A^{22} = \frac{k^2}{R^2\left(R^2 + k^2\right)} \tag{5}$$

C_0-variant coefficient of the second surface form are:

$$B_{11} = A_3\frac{\partial A_1}{\partial \varphi} = R, \quad B_{12} = B_{21} = \frac{\partial A_1}{\partial \psi} = R, \quad B_{22} = A_3\frac{\partial A_2}{\partial \psi} = R \tag{6}$$

and mixed coefficient are:

$$B_1^1 = B_1^2 = \frac{R}{R^2 + k^2}, \quad B_1^2 = B_2^2 = \frac{k^2}{R\left(R^2 + k^2\right)} \tag{7}$$

As $B_{12} \neq 0$, coordinate lines are not curve lines, all the relations of shell theory are given in tensor form. All Christoffels symbols of the first and second type equal zero as they are all the coefficients of the first fundamental form of the constant, thus co-variant derivatives are reduced to partial ones.

2.2. Deformation values

Only the displacement in direction of U_3 normal will be considered, this implying shell bending. As the cylindrical shell is continually loaded along the helix $\psi = 0$ and for the same helix boundary conditions not depending on φ coordinate are used, we assume that any value (either deformation or statistical one) independent on φ angle is distant enough from the shell ends.

Deformation in tangent plane computed by using formula

$$e_{\alpha\varphi} = \frac{1}{2}\left(u_{\alpha/\beta} + u_{\beta/\alpha}\right) - B_{\alpha\beta}u_3 \tag{8}$$

in our case, the above assumption included, equals

$$e_{11} = e_{12} = e_{21} = e_{22} = -Ru_3 \tag{9}$$

Expression for curve change and »torsion« (Eisteins convention on additional is applied)

$$K_{\alpha\varphi} = -\overline{K}_{\alpha\beta} = -\overline{K}_{\beta\alpha} = -\left[u_{3/\alpha\beta} + B^{\nu}_{\alpha/\beta}u_{\nu} + B^{\nu}_{\alpha}u_{\nu/\beta} + B^{\nu}_{\beta}u_{\nu/\alpha} - B^{\nu}_{\beta}B_{\alpha\nu}u_3\right] \tag{10}$$

are reduced, in our case under the above assumption to

$$\overline{K}_{11} = \overline{K}_{12} = \overline{u}_3, \overline{K}_{22} = \frac{d^2U_3}{d\psi^2} - u_3 \tag{11}$$

Deformation values $\overline{\rho}_{(\chi\delta)}$ are introduced, being computed by formulae

$$\overline{\rho}_{(\chi\delta)} = \rho_{(\chi\delta)} + \left(B^{\nu}_{\chi}e_{\nu\delta} + B^{\nu}_{\delta}e_{\nu\chi}\right)/2, \rho_{(\chi\delta)} = -\overline{K}_{(\chi\delta)}. \tag{12}$$

In our case $\overline{\rho}_{(\chi\delta)}$ values are equal:

$$\overline{\rho}_{(11)} = \overline{\rho}_{(12)} = 0, \overline{\rho}_{(22)} = -\frac{d^2u_3}{d\psi^2}. \tag{13}$$

2.3. Differential equation of equilibrium

Due to a U_1, U_2 displacement heglection, only one differential equation of equilibrium is used [2]:

$$\hat{M}^{(\beta\alpha)}_{|\beta\alpha} + B_{\alpha\beta}\hat{N}^{\alpha\beta} - B_{\alpha\beta}B^{\beta}_{\gamma}\hat{M}^{(\gamma\alpha)} = 0 \tag{14}$$

In our case under the introduced assumptions, in its developed form, it is:

$$d^2 \hat{M}^{(22)} / d\psi^2 - \hat{M}^{(11)} - 2\hat{M}^{(12)} - \hat{M}^{(22)} + R\left[\hat{N}^{11} + 2\hat{N}^{12} + \hat{N}^{22}\right] = 0. \tag{15}$$

2.4. Constitutive relations

Relations between statical and deformation values

$$\hat{N}^{\alpha\beta} = C\left[\nu A^{\alpha\beta}A^{\gamma\delta} + (1-\nu)A^{\alpha\gamma}A^{\beta\delta}\right]e_{\gamma\delta},\ \hat{M}^{(\alpha\beta)} = B\left[\nu A^{\alpha\beta}A^{\gamma\delta} + (1-\nu)A^{\alpha\gamma}A^{\beta\delta}\right]\bar{\rho}_{\gamma\delta}, \tag{16}$$

in our case are:

$$\hat{N}^{11} = -C\frac{R^2 + \nu k^2}{R\left(R^2 + k^2\right)^2}u_3, \qquad \hat{N}^{12} = -C\frac{(1-\nu)k^2}{R\left(R^2 + k^2\right)^2}u_3, \qquad \hat{N}^{22} = -C\frac{k^2\left(k^2 + \nu R^2\right)}{R^3\left(R^2 + k^2\right)^2}u_3 \tag{17}$$

$$\hat{M}^{11} = -B\frac{\nu k^2}{R^2\left(R^2 + k^2\right)^2}\frac{d^2u_3}{d\psi^2}, \qquad \hat{M}^{12} = 0, \qquad \hat{M}^{22} = -B\frac{k^4}{R^4\left(R^2 + k^2\right)^2}\frac{d^2u_3}{d\psi^2} \tag{18}$$

2.5. Differential equation of equlibrium according to U_3, measurement, boundary conditions and equation solving

Replacing statistical values expressed through U_3 displacement by on equilibrium equation, we have

$$\frac{d^4u_3}{d\psi^4} - \frac{k^2 + \nu R^2}{k^2}\frac{d^2u_3}{d\psi^2} + \frac{C}{B}\frac{R^2\left(R^2 + k^2\right)^2}{k^4}u_3 = 0 \tag{19}$$

This the problem of searching for the displacement field of a thin cylin dried shell loaded with a continual couple along helix has been reduced to the differential equation by u_3 with corresponding boundary conditions. The solution is being sought in the interval $\psi \in \left[0, \psi^* = 2\pi k^2/\left(R^2 + k^2\right)\right]$, where ψ^* is the helix cross-section $\varphi = 0$ with helix $\psi = 0$ along which the couple is distributed. M action is along the tangent in it collides tile the sign with the couple $2\hat{M}_{<22>}$. As the connection, betwen the physical component $\hat{M}^{(22)}$ and displacement M_3 is in the form of

$$M_{<22>} = -B\frac{k^2}{k^2\left(R^2 + k^2\right)}\frac{d^2u_3}{d\psi^2} \tag{20}$$

helix points $\psi = 0$ are considered to be joint-connected, thus boundary conditions will be for:

$$\psi = 0: \quad u_3 = 0, \qquad \frac{d^2u_3}{d\psi^2} = -\frac{R^2\left(R^2 + k^2\right)}{k^2}\frac{M}{2B} \tag{21}$$

$$\psi = \psi^*: \quad u_3 = 0, \qquad \frac{d^2u_3}{d\psi^2} = \frac{R^2\left(R^2 + k^2\right)}{k^2}\frac{M}{2B} \tag{22}$$

By introducing designation $2p = \left(k^2 + \nu R^2\right)/k^2$ and $n^2 = (C/B)\cdot R^2\left(R^2 + k^2\right)^2/k^4$ for simplicity, the solution of a typical equation that are conjugationally complex ones (n>p) can be written in the form of $\nu_1 = s + it,\quad \nu_2 = -s - it,\quad \nu_3 = s - it,\quad \nu_4 = -s + it,$, where $s = \sqrt{\frac{n+p}{2}}$, $t = \sqrt{\frac{n-p}{2}}$.

Then the solution of a differential equation has the form of

$$u_3(\psi) = e^{s\psi}\left(C_1\cos t\psi + C_2\sin t\psi\right) + e^{-s\psi}\left(C_3\cos t\psi + C_4\sin t\psi\right) \tag{23}$$

Solving the system of equation obtained from boundary conditions, constants $C_i, i = 1..4$,

$$C_1 = -\frac{\bar{u}}{4st}\cdot\frac{\sin t\psi^*}{ch s\psi^* - \cos t\psi^*}, \qquad C_2 = -\frac{\bar{u}}{4st}\left(\frac{sh t\psi^*}{ch s\psi^* - \cos t\psi^*}\right), \qquad C_3 = -C_1, \qquad C_4 = C_2 + \frac{\bar{u}}{2st} \tag{24}$$

where $\bar{u} = \frac{R^2\left(R^2 + k^2\right)}{k^2}\frac{M}{2B}$.

Thus, differential equation solution is

$$u_3(\psi) = -\frac{\bar{u}}{2st}\left[\frac{\sin t\psi^*}{ch s\psi^* - \cos t\psi^*} sh s\psi \cos t\psi + \left(sh s\psi - \frac{sh s\psi^*}{ch s\psi^* - \cos t\psi^*} ch s\psi\right)\sin t\psi\right] \tag{25}$$

The solution obtained is valid for enought from the ends of the cylinder being seized the load on the shell is not self-equilibrating, so it is necessary to prevent the movement of the shel as a solid body.

If we equal the first derivative U_3 to zero, we obtain an equation

$$tgt\psi = (Bths\psi - D)/(Cths\psi - A) \tag{26}$$

where

$$A = s(chs\psi^* - cost\psi^*), \quad B = t(chs\psi^* - cost\psi^*)$$
$$C = sshs\psi^* + t\sin t\psi^*, \quad D = tshs\psi^* - s\sin t\psi^* \tag{27}$$

out of which, for concrete values s, t, ψ^*, by an iterrative method the value ψ_m is determined for which the function $U_3(\psi)$ has an extreme value. It is evident that ψ_m is independent on the moment value M. For concrete values of cylindrical shell parameters: R=127 [mm], H=140 [mm], $k = H/2\pi = 22.2817[mm]$, E=200000 [MP_a], and shell thickness of 2,3,4 and 6 [mm], respectively, a position of the displacement value is maximum and the value of that displacement are determined, which can bee seen in Table 1.

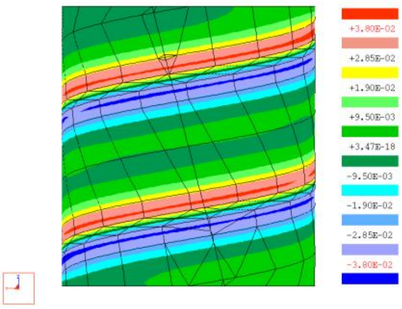

Figure 1. Radial displacement for model of cylindric shell

Table 1. Displacement values of cilindric shell

$h_c[mm]$	$\psi_m[rad]$	$u_3(\psi_m)/M[mm/N]$
6	$2{,}2583 \cdot 10^{-2}$	$0{,}9004 \cdot 10^{-5}$
4	$1{,}8284 \cdot 10^{-2}$	$2{,}0137 \cdot 10^{-5}$
3	$1{,}5840 \cdot 10^{-2}$	$3{,}5803 \cdot 10^{-5}$
2	$1{,}2941 \cdot 10^{-2}$	$8{,}0623 \cdot 10^{-5}$

3. COMPARISON OF THEORETICAL AND NUMEICAL RESULTS

To compare theoretical and numerical results the programme package Pro/MECHANICA capable of defining a continual couple along helix is used. In Figure 1. results of displacement in radial direction for model of cylindric shell with the same parameters from the previous chapter and thicklness of 3 [mm] are shown. The shell along its helicoidal line is loaded by a continual moment couple M=1000 [Nmm/mm] [3,4].

4. CONCLUSION

By solving the homogenous linear differential equation with constant coefficients of displacement in the direction of the normal to the cylinder, a position of points on cylindric shell in which the value of displacement is maximum and the value of that displacement are determined. It is show that the position of these points is independent on the intensity of a continual couple. Comparing the results with those ones obtained by the programme package for MKE Pro/MECHANICA, an agreement of theoretical and numerical analyses is obtained, introduced assumptions of the efficiency of the calculation methodology of this model of cylindrical shell is proved.

5. REFERENCES

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