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APLICATION OF CONTROL TERMS P1 AND P2 TO ESTIMATION OF ROTATION CURVE OF STARS

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ABSTRACT

As it is the well known, the control terms P1 and P2 are used in control theory for description of the dynamics of control objects. The control terms P1 and P2 describe linear system of the first and second order, respectively. In this paper it has been shown that these terms can also be used in the estimation of the rotation curve of the stars around galaxy. Following the related observation information, control terms P1 estimates the rotation curve related to Dark Matter, while control term P2 estimates the rotation curve related to Total Matter (Visible Matter + Dark Matter). On that way one obtains the analytical models for rotation curves that can be used for calculation of the mass of Dark and Total Matters.

Keywords: rotation curves, dark matter, proportional control terms P1 and P2.

1. INTRODUCTION

As it is the well known, the predicted and observed rotation curve of a typical spiral galaxy are different. The predicted rotation curve is slowing down when we run out of the stars (visible matter) in the galactic disc. On the other hand, the observed rotation curve keeps the constant velocity when we run out of the stars in the galactic disc. Dark matter can explain this 'flat' appearance of the velocity curve out to a large radius [1,2]. Dark matter cannot be seen directly with telescopes because it neither emits nor absorbs light or other electromagnetic radiation at any significant level. Instead, the existence and properties of dark matter are inferred from its gravitational effects on visible matter, radiation, and the large-scale structure of the universe. Unfortunately, the composition of the dark matter is unknown.

The problem is to construct analytical models that can be useful in the estimation of the observed rotation curve of typical spiral galaxies. In this paper we proposed the procedure for determination of the analytical models for rotation curves that can estimate the observed rotation curve of a typical spiral galaxy. In that sense, we started with the two well known control terms P1 and P2 that usually are used in control theory for description of the dynamics of control objects. The control terms P1 and P2 describe linear system of the first and second order, respectively. In this paper it has been shown that these terms can also be used in the estimation of the rotation curve of the stars around galaxy. Following the related observation information, control term P1 estimates the rotation curve related to Dark Matter, while control term P2 estimates the rotation curve related to Total Matter (Visible Matter + Dark Matter). On that way one obtains the analytical models for rotation curves that can be used, among the others, for calculation of the mass of Dark and Total Matters in the typical spiral galaxies.

2. ANALYTIC ESTIMATION OF STARS ROTATION CURVES ABOUT GALAXY

In order to derive the analytic estimation equation for stars velocities about galaxy, one can consider a linear general second-order Input-Output (IO) system with $q_1 = q_2 = 0$ (P_2 system) and a linear general first-order Input-Output (IO) system with $q_1 = 0$ (P_1 system), given in the following forms [3]:

$$P_2 \quad \rightarrow \quad \ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = q_0 u, \qquad P_1 \quad \rightarrow \quad \dot{y} + p_0 y = q_0 u. \tag{1}$$

In these equations u is an input variable, y is an output variable, ω_n is a natural frequency, ζ is a dimensionless damping ratio, and $p_0 \neq 0$ and $q_0 \neq 0$ are related coefficients. If in the P_2 system $u(t) = 0$, then the relation P_2 in (1) describes the so-called damped harmonic oscillator.

Let the systems given by (1) are subjected to a step input function $u(t) = u_0 = \text{const.}$, where in P_2 system we have $y(0) = \dot{y}(0) = 0$, $\omega_n > 0$ and $0 \leq \zeta < 1$ and in P_1 system we have $y(0) = 0$. For those cases the systems (1) have the well known solutions:

$$P_2 \rightarrow y(t) = K_p\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \varphi)\right], \quad K_p = u_0\frac{q_0}{\omega_n^2}, \quad \omega_d = \omega_n\sqrt{1-\zeta^2},$$
$$\varphi = \arccos(\zeta), \ 0 \leq \varphi < \frac{\pi}{2}, \quad P_1 \rightarrow y(t) = K_{p1}\left[1 - e^{-t/\tau}\right], \ K_{p1} = \frac{q_0}{p_0}, \ \tau = \frac{1}{p_0}. \tag{2}$$

Here K_p is the proportional parameter, ω_d is the damped frequency and φ is the phase angle in P_2 system, while K_{p1} is the proportional parameter and τ is the time constant in P_1 system.

2.1. Proposition 1.

Let the output and input variables, $y(t)$, and $u(t)$, of the linear general second-order (P_2) and first-order (P_1) IO systems (1) are replaced by the star velocity $v(r)$ in P_2, or $v_d(r)$ in P_1, and input variable $u(r)$, respectively, as functions of the radius, r:

$$P_2 \rightarrow t \leftrightarrow r, \ y(t) \leftrightarrow v(r), \ u(t) \leftrightarrow u(r), \quad P_1 \rightarrow t \leftrightarrow r, \ y(t) \leftrightarrow v_d(r), \ u(t) \leftrightarrow u(r). \tag{3}$$

Here $v(r)$ is the star velocity related to the total matter (visible + dark), $v_d(r)$ is the star velocity related to the dark matter, and radius r determines the distance between a star and the centre of a galaxy. Now, including (3) into the relation (1), the dynamics of the stars velocities around the galaxy can be approximately described by the following equations:

$$P_2 \quad \rightarrow \quad \ddot{v}(r) + 2\zeta\omega_n\dot{v}(r) + \omega_n^2 v(r) = q_0 u(r), \quad \dot{v}(r) = \frac{dv}{dr}, \quad \ddot{v}(r) = \frac{d^2v}{dr^2},$$
$$P_1 \quad \rightarrow \quad \dot{v}_d(r) + p_0 v_d(r) = q_0 u(r), \quad \dot{v}_d(r) = \frac{dv_d}{dr}, \quad 0 \leq r \leq r_e. \tag{4}$$

Here r_e is the radius region in which the equations (4) are valid. Further, let the systems, given by (4), are subjected to a step input function $u(r) = u_0 = \text{const.}$, where in P_2 system we have $v(0) = \dot{v}(0) = 0$, $\omega_n > 0$ and $0 \leq \zeta < 1$, while in P_1 system we have $v_d(0) = 0$. For those cases the solutions of the systems (4) are given by the relations:

$$P_2 \ \rightarrow \ v(r) = K_p\left[1 - \frac{e^{-\zeta\omega_n r}}{\sqrt{1-\zeta^2}}\sin(\omega_d r + \varphi)\right], \quad P_1 \ \rightarrow \ v_d(r) = K_{p1}\left[1 - e^{-r/\tau}\right]. \tag{5}$$

The solutions $v(r)$ and $v_d(r)$ in (5) describe the approximations of the stars velocities around related galaxy.

2.2. Proof of the Proposition 1. In order to prove the Proposition 1 for P_2 system, one should determine, by the observation and simulation, the set of parameters (K_p, ω_n, ζ) for each galaxy and

to calculate the parameters ω_d and φ by using (2). In order to prove the Proposition 1 for P_1 system, one should determine, by the observation and simulation, the parameters K_{p1} and τ for each galaxy. Further, the obtained velocities curves $v = f(r)$ and $v_d = f_d(r)$ from (5) should be compared to the observed velocities curves $v_{ob} = f_{ob}(r)$ and $v_{dob} = f_{dob}(r)$ for the related galaxy.
The estimation of the approximation of the stars rotational curve around the galaxy has been proved by the three galaxies: the Andromeda galaxy, the NGC 3198 galaxy and our Milky Way galaxy. Because of the limited space, here has been presented the Proof of the Proposition 1 only for the NGC 3198 galaxy. The observed rotation curves for the stars in the disk of the NGC 3198 galaxy is taken from [2] and shown by Fig. 1.

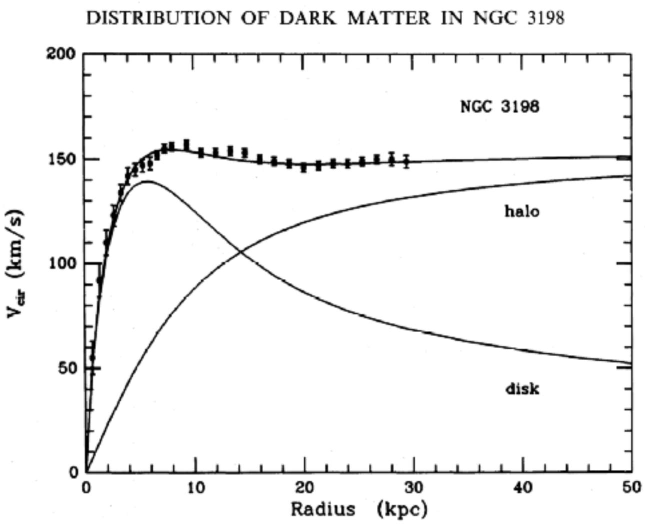


Fig. 1: The observed rotation curves (v_{ob}) for the stars in the disk of the NGC 3198 galaxy [2].

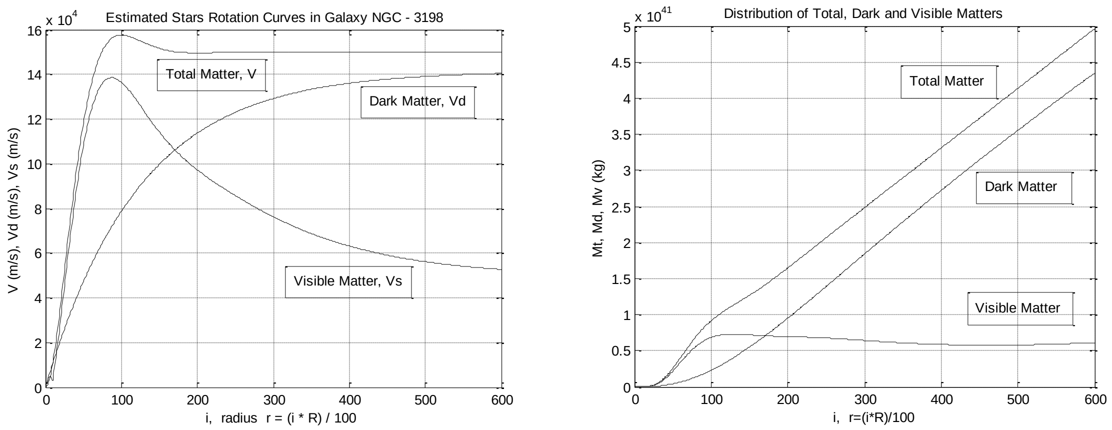
As it is well known, the stars in this galaxy extend out only to 10 kpc [1,2]. But from Fig. 1 one can see that the rotation curve remains flat out to 30 kpc. There must be something besides the stars dominating the mass of the galaxy. The curve labelled by "disk" indicates the expected rotation curve due to the stars (visible matter) in the galaxy. The curve labelled by "halo" indicates the rotation curve due to the "dark matter halo" of the galaxy. Unfortunately, the composition of the "dark matter halo" is unknown. From Fig. 1 we can see that at large radial distance the rotation curve is flat. This means that the rotational velocity v = v_f = constant (about 150 km/sec). Further, from this figure one can directly estimate for P_2 system the maximal velocity v_{max} at the radius $r_{v_{max}}$ and the proportional parameter $K_p = v_f$. For P_1 system one can estimate the proportional parameter K_{p_1} which is equal to maximal velocity related to dark matter $v_{d\,max}$ and constant τ. For visible mater we estimate the maximal velocity $v_{v\,max}$ at the radius $r_{v_{v\,max}}$ and velocity v_{se} at the radius r_e. Finally, we estimate the radius region r_e in which the equations (4) and (5) are valid:

$$\begin{gathered} v_{max} = 157\cdot 10^3\,\text{m/s} \;\leftrightarrow\; r_{v_{max}} = R = 2.453\cdot 10^{20}\,\text{m},\;\; K_p = v_f = 150\cdot 10^3\,\text{m/s}, \\ r_e = 1.471\cdot 10^{21}\,\text{m}, \quad K_{p_1} = v_{d\,max} = 141\cdot 10^3\,\text{m/s}, \quad \tau = 3.016\cdot 10^{20}\,\text{m}, \\ v_{v\,max} = 139\cdot 10^3\,\text{m/s} \;\leftrightarrow\; r_{v_{v\,max}} = 2.158755\cdot 10^{20}\,\text{m}, \quad v_{se} = 52\cdot 10^3\,\text{m/s}. \end{gathered} \tag{6}$$

Using the parameters from (6) and the related computer simulation of velocity equation $v(r)$ from (5), one can estimate the following parameters, valid for the NGC 3198 galaxy:

$$\zeta = 0.689, \qquad \omega_n = 1.766989\cdot 10^{-20}\,\text{s}^{-1}, \qquad \varphi = 0.810687\,\text{rad}. \tag{7}$$

Applying the parameters from (6) and (7), we obtain the estimated rotation curves v, v_d and v_s for the stars in the disk of the NGC 3198 galaxy, shown in the Fig. 2a.



a) b)

Fig. 2: The estimated rotation curves (v, v_d, v_s) (a), and (b) distribution of total, dark and visible matters for the stars in the disk of the NGC 3198 galaxy.

Table 1. Comparison between observed and estimated rotation curves, Fig. 1 and Fig. 2a, respectively, has been presented by five control points.

Velocities	Radius r	Observed values	Estimated values
Max. Velocity V_{max}	$2.453 \cdot 10^{20} m$	$157 \cdot 10^3 m/s$	$157.5689 \cdot 10^3 m/s$
Flat velocity V_f	$1.471878 \cdot 10^{21} m$	$150 \cdot 10^3 m/s$	$150.000049 \cdot 10^3 m/s$
Dark Matter Max. Veloc. V_{dmax}	$1.471878 \cdot 10^{21} m$	$141 \cdot 10^3 m/s$	$140.4252 \cdot 10^3 m/s$
Visible Matter Max. Veloc. V_{max}	$2.158755 \cdot 10^{20} m$	$139 \cdot 10^3 m/s$	$138.5435 \cdot 10^3 m/s$
Visible Matter Velocity V_{se}	$1.471878 \cdot 10^{21} m$	$52 \cdot 10^3 m/s$	$52.7327 \cdot 10^3 m/s$

The comparison between observed and estimated rotation curves, Fig. 1 and Fig. 2a, respectively, and between observed and estimated data in the Table 1 shows an acceptable estimation of the observation data. Thus, on that way, the Proof of the Proposition 1 has been finished for the NGC 3198 galaxy. Using the well known relations from the general relativity [4] we can calculate the total matter M_t, dark matter M_d and visible matter M_v : $M_t = r v^2 / G$, $M_d = r v_d^2 / G$, $M_v = r v_s^2 / G$. Here G is the Newtonian gravitational constant. Applying those relations we obtain the distribution of the total, dark and visible matters in the disk of the galaxy NGC 3198, shown on the Fig. 2b.

3. CONCLUSION

In this paper it has been shown and proved that the control terms P1 and P2 can be used in the estimation of the rotation curve of the stars around galaxy. Following the related observation information, control terms P1 estimates the rotation curve related to Dark Matter, while control term P2 estimates the rotation curve related to Total Matter (Visible Matter + Dark Matter). On that way one obtains the analytical models for rotation curves that can be used, among the others, for calculation of the mass of Dark and Total Matters in the typical spiral galaxies. In the future work the influence of Dark Matter will be included in the calculation of the motion of nanorobotic rockets [5].

4. REFERENCES

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