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ANALYTICAL SOLUTION FOR ANGLE-PLY PLATES USING FIRST ORDER SHEAR DEFORMATION PLATE THEORY AND NAVIER SOLUTION TYPE SS-2

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ABSTRACT
*This paper presents a solution for static analysis of simply supported rectangular composite plates based on First order shear deformation plate theory (**FSDT**). Linear equations of motion **FSDT** theory are derived and then in MathLab shown a effect of shear deformation on nondimensionalized natural frequencies of simply supported (SS-2) symmetric angle-ply rectangle plate, transverse shear deformation on nondimensionalized critical buckling loads of simply supported (SS-2), antysymmmetic angle-ply rectangle plate and transverse shear deformation on nondimensionalized maximum transverse deflections and stresses of simpy supported (SS-2).*
Keywords: Composite plate, FSDT, Shear stress.

INTRODUCTION
This paper presents a solution for static analysis of simply supported rectangular composite plates based on First order shear deformation plate theory (FSDT). With the help of MathLab shows an example of two-layered angle-ply rectangular plate, simply supported at the edges. The layers are at an angle θ = 45 ˚ / -45 ˚. Such procedures are obtained solutions are in the range of theoretical values.

1. GOVERNING EQUATIONS
In First order shear laminate plate theory is not valid third assumption Kirchhoff's classical theory of plates. This fact in FSDT introduces shear stresses. Other assumptions of classical plate theory remain valid. Displacement field of rectangular plates based on the classical plate theory, including the effects of shear deformation, we can write:

$$u(x,y,z,t) = u_o(x,y,t) + z \cdot \phi_x(x,y,t) \quad ...(1)$$

$$v(x,y,z,t) = v_o(x,y,t) + z \cdot \phi_y(x,y,t) \quad ...(2)$$

$$w(x,y,z,t) = w_o(x,y,t) \quad ...(3)$$

$$\phi_x = \frac{dw_0}{dx} \quad ...(4)$$

$$\phi_y = \frac{dw_0}{dy} \quad ...(5)$$

, $u_o(x,y,t)$, $v_o(x,y,t)$, $w_o(x,y,t)$ denote the corresponding mid-plane displacements in the x, y, z direction, and φ_x, φ_y rotation are perpendicular to the median plane of x and y direction.

2. NUMERICAL PROCEDURE

2.1 Navier's boundary conditions, Type SS-2

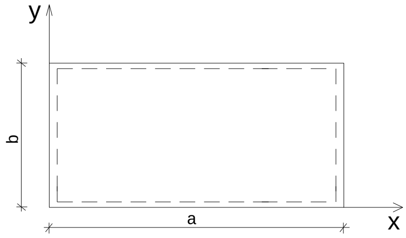

Figure 1. Preview of asymmetric Angle-Ply rectangular plate, simply supported at the edges, Type SS-2

The boundary conditions of Navier's solution:

for x=0,a $\quad u_0=w_0=\phi_y = 0 \quad N_{xy} = M_{xx} = 0$ ...(6)

for y=0,b $\quad v_0=w_0=\phi_x = 0 \quad N_{xy} = M_{yy} = 0$ ...(7)

Based on the above mentioned boundary conditions to the Navier's solution type SS-2, displacement field of rectangular plate, which is the subject of this paper:

$$u_0(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} U_{mn}(t) sin\alpha x\, cos\beta y \qquad ...(8)$$

$$v_0(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} V_{mn} cos\alpha x\, sin\beta y \qquad ...(9)$$

$$w_0(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} W_{mn}(t) sin\alpha x sin\beta y \qquad ...(10)$$

$$\phi_x(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} X_{mn}(t) cos\alpha x sin\beta y \qquad ...(11)$$

$$\phi_y(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} Y_{mn}(t) sin\alpha x\, cos\beta y \qquad ...(12)$$

This requirement applies to general ortrotropan single-layer laminate for symmetrical laminate with layers of specially ortrotropnih and antisymmetric angle-ply laminate.

Linear equations of motion FSDT theory:

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} -(A_{11}\alpha^2 + A_{66}\beta^2)U_{mn} - (A_{12} + A_{66})\alpha\beta V_{mn} + (3B_{16}\alpha^2\beta + B_{26}\beta^2)W_{mn}$$
$$-I_0\ddot{U}_{mn}] sin\alpha x\, cos\beta y = 0 \qquad ...(13)$$

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} -(A_{12}+A_{66})\alpha\beta U_{mn} - (A_{66}\alpha^2 + A_{22}\beta^2)V_{mn} + (B_{16}\alpha^3 + 3B_{26}\alpha\beta^2)W_{mn}$$
$$-I_0\ddot{V}_{mn}]cos\alpha x\ sin\beta y = 0 \qquad \ldots (14)$$
$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} [(3B_{16}\alpha^2\beta + B_{26}\beta^3)U_{mn} + (B_{16}\alpha^3 + 3B_{26}\alpha\beta^2)V_{mn}$$
$$-(D_{11}\alpha^4 + 2(D_{12}+2D_{66})\alpha^2\beta^2 + D_{22}\beta^4)W_{mn}$$
$$-(\alpha^2\widehat{N}_{xx} + \beta^2\widehat{N}_{yy})W_{mn} - (I_0 + I_2(\alpha^2+\beta^2))\ddot{W}_{mn}]sin\alpha x\ cos\beta y$$
$$= -q(x,y) \qquad \ldots (15)$$
$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \left((D_{11}\alpha^2 + 2D_{66}\beta^2 + KA_{55})X_{mn} + (D_{12}+D_{66})\alpha\beta Y_{mn}\right) + I_2\alpha\ddot{W}_{mn}$$
$$= 0$$
$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left((D_{12}+D_{66})\alpha\beta X_{mn} + (D_{66}\alpha^2 + 2D_{22}\beta^2 + KA_{44})Y_{mn} + I_2\beta\ddot{W}_{mn}\right) = 0 \qquad (16)$$

The problem is described with five differential equations of second order with five variables $U_{m,n}$, $V_{m,n}$, $W_{m,n}$, T_x, T_y.

3. DETERMINATION OF STRESS, BUCKLING AND VIBRATION

Transverse stresses may be derived from the equations of equilibrium elastic 3-D, where:

$$\sigma_{xz}^{(k)}(x,y,z) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[(z-z_k)\bar{\mathcal{A}}_{mn}^{(k)} + \frac{1}{2}(z^2 - z_k^2)\bar{B}_{mn}^{(k)}\right] + \sigma_{xz}^{(k-1)}(x,y,z_k) \qquad \ldots (17)$$

$$\sigma_{yz}^{(k)}(x,y,z) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[(z-z_k)\bar{C}_{mn}^{(k)} + \frac{1}{2}(z^2 - z_k^2)\bar{D}_{mn}^{(k)}\right] + \sigma_{yz}^{(k-1)}(x,y,z_k) \qquad \ldots (18)$$

With buckling analysis we assume that the load acting on the plane. By eliminating movement in the plane (Umn, Vmn) we get:

$$N_0 = \frac{1}{\alpha^2 + k\beta^2}\left(\hat{c}_{33} - \frac{\hat{c}_{34}\hat{c}_{55} - \hat{c}_{35}\hat{c}_{45}}{\hat{c}_{44}\hat{c}_{55} - \hat{c}_{45}\hat{c}_{45}}\hat{c}_{34} - \frac{\hat{c}_{44}\hat{c}_{35} - \hat{c}_{45}\hat{c}_{34}}{\hat{c}_{44}\hat{c}_{55} - \hat{c}_{45}\hat{c}_{45}}\hat{c}_{35}\right) \qquad \ldots (19)$$

Equations frequencies specially orthotropic laminates and laminates antisimtričnih looks like:

$$\omega^2 = \frac{1}{\hat{m}_{33}}\left(\hat{c}_{33} - \frac{\hat{c}_{13}\hat{c}_{22} - \hat{c}_{23}\hat{c}_{12}}{\hat{c}_{11}\hat{c}_{22} - \hat{c}_{12}\hat{c}_{12}}\hat{c}_{13} - \frac{\hat{c}_{11}\hat{c}_{23} - \hat{c}_{12}\hat{c}_{13}}{\hat{c}_{11}\hat{c}_{22} - \hat{c}_{12}\hat{c}_{12}}\hat{c}_{23}\right) \qquad (20)$$

4. NUMERICAL EXAMPLES

Enter the number of layers is 2, entered thicknesses are 0.05 wide layers are loaded a=1 i b=1. The characteristics of the first layer are as follows: E_1 =20000[MPa], E_2 =800[Mpa], G_{12} =401[Mpa], G_{23}=80[Mpa], G_{13} =401[Mpa], u_{12}=0.25, u_{21}=0.25, Θ_0=-45°. Characteristics of the second layer thickness b=1 are: E_1 =20000[MPa], E_2 =800[Mpa], G_{12} =401[Mpa], G_{23}=80[Mpa], G_{13} =401[Mpa], u_{12}=0.25, u_{21}=0.25, Θ_0=45°.

4.1 Analytical solution for rectangular laminated plates by FSDT

For such a loaded layers, for odd values of m, n = 1 obtained stress values (21), (22), the critical buckling force (23) and vibration (24) Based on the above equations are obtained:

$$\bar{\sigma}_{xx} = \sigma_{xx} * \frac{h^2}{b^2 q_0} = 33.52 * \frac{0.1^2}{1^2 * 1} = 0.3352 \quad \approx \quad 0.3476 \qquad \text{...}(21)$$

$$\bar{\sigma}_{xy} = \sigma_{xy} * \frac{h^2}{b^2 q_0} = 40.125 * \frac{0.1^2}{1^2 * 1} = 0.4012 \quad \approx \quad 0.4274 \qquad \text{...}(22)$$

$$\bar{N} = N_{cr} * \frac{a^2}{E_2 * h^3} = 5327.9 * \frac{1^2}{800 * 10^3 * 0.1^3} = 6.659 \quad \approx \quad 6.115 \;\text{....}(23)$$

$$\bar{w} = w * \frac{a^2}{h} \sqrt{\frac{\rho}{E_2}} = 10{,}25 * \frac{1^2}{0.1} \sqrt{\frac{10}{800}} = 11{,}459 \quad \approx \quad 10{,}895 \qquad \text{....}(24)$$

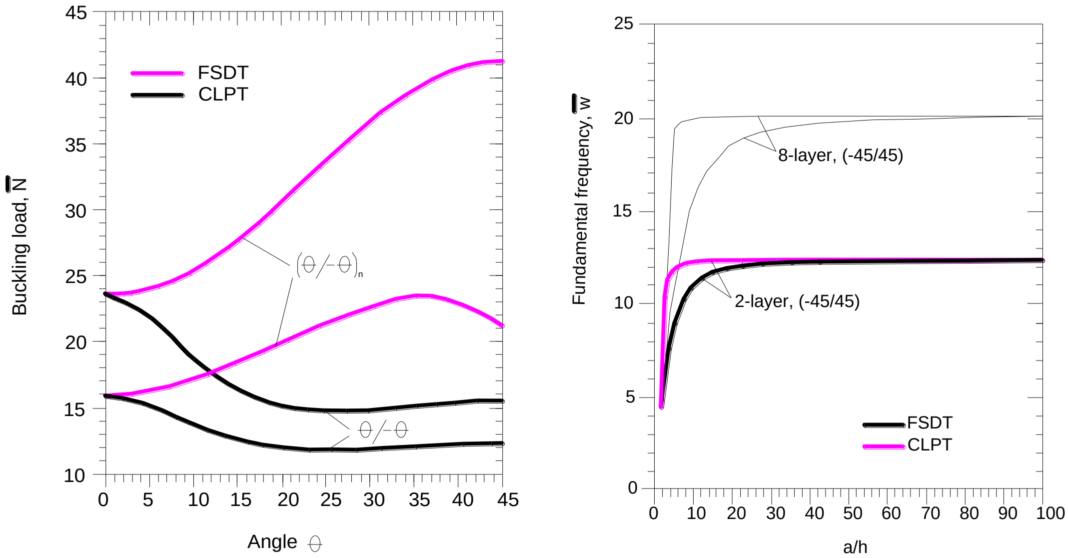

Figure 2. Diagrams theoretically expected values of critical buckling force N frequency of antisymmetric angle-ply rectangular plate, Type SS-2.

5. REFERENCES

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