

## FATIGUE CRACK SHAPE DEVELOPMENT

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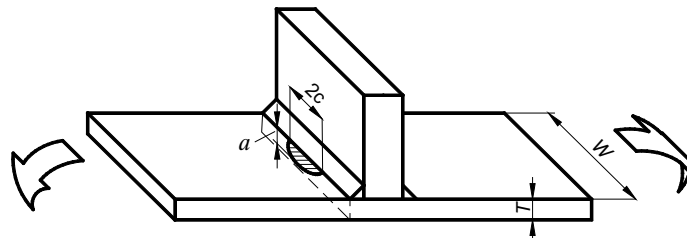
### ABSTRACT

*The big part of fatigue life is spent in fatigue crack propagation. Fatigue cracks emanating from stress concentrators (weld toe in this case) have semi-elliptical shape. Cracks change their shape (aspect ratio  $a/2c$ ) during their growth. An assumed constant value of aspect ratio has a big influence on stress intensity factor and crack propagation life. The realistic development of fatigue crack shape during crack propagation is predicted in this paper, applying the crack growth rules separately to the depth and length directions of the crack assuming a semi-elliptical shape between these principal directions. This prediction is performed by the computer program based on step by step solving differential equation describing fatigue crack growth.*

**Keywords:** welded joint, fatigue crack shape development, aspect ratio

### 1. INTRODUCTION

The fatigue fracture of structural details subjected to cyclic loads mostly occurs at a critical cross section with stress concentration. In a welded joint (Fig.1), fatigue crack initiate at the weld toe and propagates through the main plate to a final fracture.



*Figure 1. Welded joint subjected to bending loading*

Driving force of this process is stress intensity factor (SIF). In this paper SIF is determined by using solution [1,2] for surface semielliptical crack in welded joint subjected to bending loading:

$$\Delta K = Y\Delta\sigma\sqrt{\pi a} \quad (1)$$

where  $\Delta K = \Delta\sigma\sqrt{\pi a}$  is the solution for a central crack of size  $2a$  in an infinite plate subjected to the remote uniform tensile stress  $\Delta\sigma$ . The total correction factor  $Y$  that modifies the value of  $\Delta K$  of idealized case, in order take account of the effects of finite width  $f_w$ , elliptical crack front  $\Phi$ ,  $M_k$  local stress concentration and  $M_b$  for crack in flat plate subjected to bending loading.

$$Y = M_{kb}M_b f_w \quad (2)$$

where

$$M_{kb} = f_1\left(\frac{a}{T}, \frac{a}{c}\right) + f_2\left(\frac{a}{T}\right) + f_3\left(\frac{a}{T}, \frac{L}{T}\right) \quad (3)$$

$$M_b = HM_m \quad (4)$$

$M_m$  is factor from solution for membrane loading;  $L$  is attachment footprint width.

$$M_m = f\left(\frac{a}{T}, \frac{a}{c}, \theta\right) / \Phi \quad (5)$$

$$\Phi = f_4\left(\frac{a}{c}\right) \quad (6)$$

$$H = f_5\left(\frac{a}{T}, \frac{a}{c}, \theta\right) \quad (7)$$

Calculated values of SIF in deepest and surface points are shown in Fig.2. For a big aspect ratio e.g.  $a/c=1.0$ , total correction factor  $Y$  in surface point is significantly bigger than the one in deepest point; this fact produces faster fatigue crack growth in surface direction than in depth direction and decrease of aspect ratio. For a small aspect ratio e.g.  $a/c=0.1$ , total correction factors in surface and deepest point are approximately equal (until  $a/T=0.3$ ), but  $c$  is significantly bigger than  $a$ , so the last member (square root) in eq.1. is bigger and causes faster fatigue crack growth in surface direction and the decrease of aspect ratio (Fig.4).

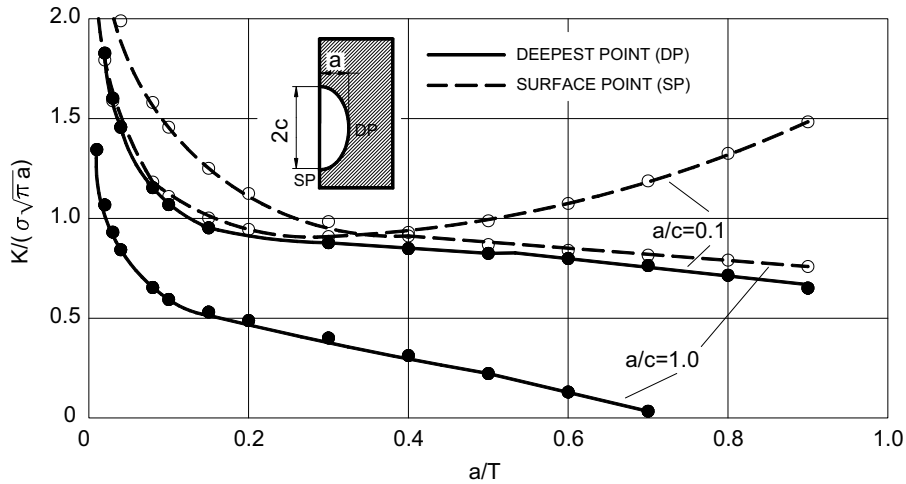


Figure 2. Relative stress intensity factor for semi-elliptical surface crack at weld toe of one-sided transverse attachment joint in deepest and surface points

## 2. CRACK SHAPE DEVELOPMENT

### 2.1. Crack propagation model

The crack propagation rate is determined by using the Paris equation [3]:

$$\frac{da}{dN} = C(\Delta K)^m \quad (8)$$

where

- $a$  = crack depth
- $N$  = number of cycles
- $da/dN$  = crack growth rate,
- $\Delta K$  = range of stress intensity factor,
- $C$  and  $m$  = material constants.

Average material properties, for steel, were assumed:  $m = 3$ ,  $C = 4.9 \cdot 10^{-12}$ , with  $\Delta K$  in units of  $\text{MPa}\sqrt{\text{m}}$  and  $da/dN$  in units of  $\text{m}/\text{cycle}$ , threshold stress intensity factor  $\Delta K_{th} = 4 \text{ MPa}\sqrt{\text{m}}$ .

## 2.2. Crack shape

Numerical integration of eq.8. was carried out step-by-step using Runge –Kutta method [4]. For small consecutive increments in number of cycles  $\Delta N=100$  was calculated increment of crack in depth and surface direction and re-calculated new aspect ratio. Calculated values of SIF in deepest (DP) and surface (SP) point during crack growth are shown in Fig.3 (in this example initial value of aspect ratio is  $a/c=1.0$ ). The SIF value in surface point is bigger than that one in deepest point. It implies that the crack growth rate along the surface will be bigger than in depth direction. Due to this facts aspect ratio decreases (Fig.4).

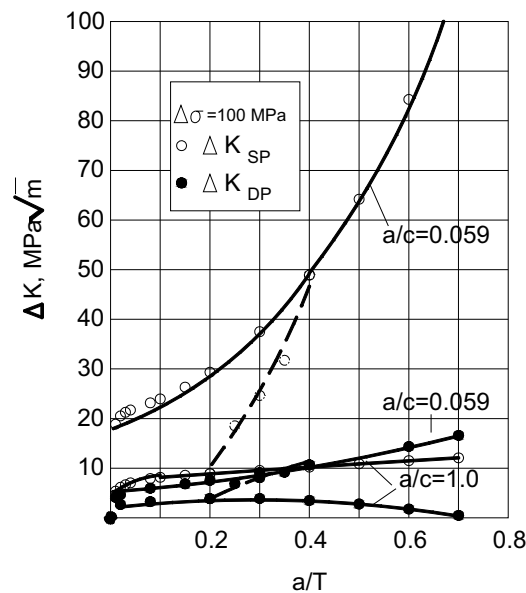


Figure 3. SIF in deepest point and surface point for constant aspect ratio (non-realistic process)  $a/c=1.0$  and  $a/c=0.059$ (—) and realistic process where aspect ratio varies during crack growth (---)

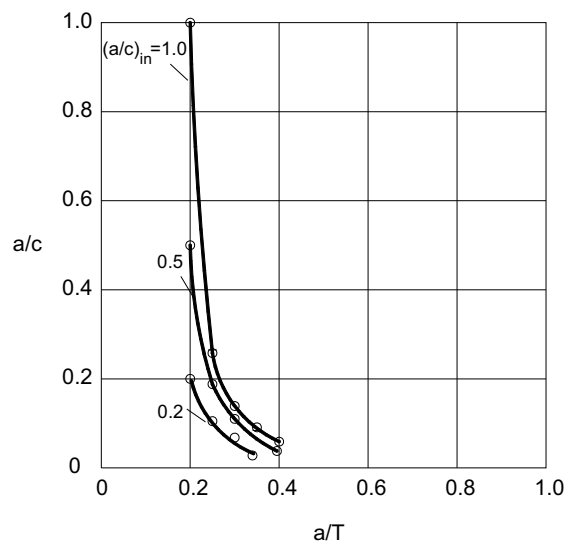


Figure.4. Fatigue crack shape development during crack growth for various values of initial crack aspect ratio  $(a/c)_{in}$

This decrease of crack aspect ratio is also illustrated by the crack fronts ( $\Delta\sigma=100$  MPa) in Fig.5 (designation explained in Table 1).

Table 1. Designation from Fig.5. and its explanation

Crack front	1		2		3	
Figure	(a/c) <sub>in</sub>	N	a/c	N	a/c	N
5a	1.0	0	0.35	500000	0.10	1000000
5b	0.5	0	0.14	500000	0.05	800000
5c	0.2	0	0.05	500000		

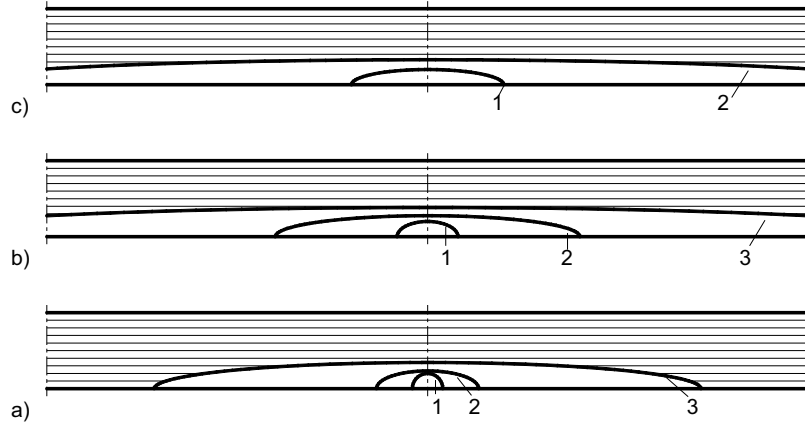


Figure 5. Crack fronts in welded joints subjected to bending loading obtained by using computer simulation

### 3. CONCLUSIONS

In welded joints subjected to bending loading fatigue crack aspect ratio decreases rapidly during crack growth. This knowledge and diagrams presented in this paper could be useful in calculation of SIF, fatigue crack growth rate and fatigue life of welded joints subjected to bending loading. Also the last figure could help in post-fracture analysis of fatigue fracture surface.

### 4. REFERENCES

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