

## MULTI-RESPONSE OPTIMIZATION OF BALL-END MILLING PARAMETERS USING THE TAGUCHI-BASED GREY RELATIONAL ANALYSIS

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### ABSTRACT

*This paper presents an approach for optimization of machining parameters with multi-response outputs using design of experiment in ball-end milling. During the ball-end milling of hardened steel, process performance indicators such as surface roughness, material removal rate and resultant cutting force were measured. The process parameters which are spindle speed, feed per tooth, axial depth of cut and radial depth of cut were simultaneously optimized by the Taguchi-based Grey relational analysis. Experiments are designed and conducted based on Taguchi's  $L_{25}$  orthogonal array design.*

*Based on grey relational grade value, optimum levels of parameters have been identified by using response table and response graph and the significant contributions of controlling parameters are estimated using analysis of variances (ANOVA). Confirmation test is conducted for the optimal machining parameters to validate the test result.*

**Keywords:** ball-milling, multi-response optimization, Taguchi-based Grey relational analysis

### 1. INTRODUCTION

In this study, the effect of ball-end milling parameters on the resultant cutting force, the surface roughness and the material removal rate (MRR) are reported using the Taguchi-based Grey relational analysis. This approach can solve multi-response optimization problem simultaneously. In this study the Taguchi  $L_{25}(5^6)$  orthogonal array was applied to plan the experiments on ball-end milling process. Four controlling factors including spindle speed  $n$ , axial depth of cut  $a_p$ , radial depth of cut  $a_e$  and feed per tooth  $f_z$  with five levels for each factor were selected.

The Grey relational analysis is then applied to examine how the cutting factors influence the resultant cutting force  $F_R$ , the surface roughness  $R_a$  and the material removal rate  $Q$ . An optimal parameter combination was then obtained. Additionally, an analysis of variance (ANOVA) was also utilized to examine the most significant factors for the  $F_R$ ,  $R_a$  and  $Q$  in the ball-end milling process.

### 2. TAGUCHI-BASED GREY RELATIONAL ANALYSIS

The integrated the Taguchi-based Grey relational analysis combines the algorithm of Taguchi method and Grey relational analysis to determine the optimum process parameters for multiple responses.

This approach converts a multiple-response process optimization problem into a single response optimization situation. The single objective function is the overall grey relational grade. Higher grey relational grade means that the corresponding parameter combination is closer to the optimal. The optimal parametric combination is then evaluated considering the overall grey relational grade by using Taguchi method. The highest grey relational grade is the rank of 1. The equations needed to apply Taguchi method and Grey relational analysis are given in Table 2.

Table 2. Equations for Taguchi method and Grey relational analysis

Taguchi method		Grey relational analysis	
S/N ratio	The S/N ratio based on the "larger-the-better" (L-T-B) criterion for overall grey relational grade was calculated using equation:  $S/N = \eta = -10 \log \left( \frac{1}{n} \sum_{i=1}^n \frac{1}{y_i^2} \right)$	Grey relational generation	The normalized data processing for $F_R$ and $R_a$ corresponding to "smaller-the-better" (S-T-B) criterion can be expressed as:  $x_i(k) = \frac{\max y_i(k) - y_i(k)}{\max y_i(k) - \min y_i(k)}$  $i=1,2,\dots,n$ , n is the number of experimental runs in Taguchi orthogonal array $L_{25}$ ( $n=25$ ); $k=1,2,\dots,m$ , m is the number of responses, in the present work the resultant force, the surface roughness and the material removal rate are selected, then $m=3$ ; $x_i(k)$ is the value after the grey relational generation; $\min y_i(k)$ is the smallest value of $y_i(k)$ for the $k^{\text{th}}$ response and $\max y_i(k)$ is the largest value of $y_i(k)$ for the $k^{\text{th}}$ response
		Grey relational coefficient	$\xi_i(k) = \frac{\Delta_{\min} + \psi \cdot \Delta_{\max}}{\Delta_{0i}(k) + \psi \cdot \Delta_{\max}}, 0 \leq \xi_i(k) \leq 1,$ $\Delta_{0i}(k) =  x_0(k) - x_i(k) $ is difference of the absolute value between $x_0(k)$ and $x_i(k)$ ; $\Delta_{\min}$ and $\Delta_{\max}$ are respectively the minimum and maximum values of the absolute differences; $\psi$ is the distinguishing coefficient, $0 \leq \psi \leq 1$ - $\psi = 0.333$ for $F_R$ , $\psi = 0.556$ for $R_a$ and $\psi = 0.111$ for $Q$ using AHP method (In this case importance has been given to roughness value [5].)
		Grey relational grade	$\gamma_i = \frac{1}{m} \sum_{k=1}^m \xi_i(k)$
		Estimated Grey relational grade	$\hat{\gamma} = \gamma_m + \sum_{i=1}^q (\gamma_i - \gamma_m),$ $\gamma_m$ - the total mean Grey relational grade; $\gamma_i$ - the mean Grey relational grade at the optimal level; q-the number of the main design parameters that affect the quality characteristic

### 3. EXPERIMENTAL PROCEDURE

The experimental work was carried out at the company "ELMETAL" doo in Senta (Serbia). The experiments were conducted on vertical machining centre type "HAAS VF-3YT" in dry condition, using a carbide coated (TiAlN-T3) ball-end mill with Ø6 mm diameter ("EMUGE FRANKEN" type 1877A). All experiments were carried out using hardened steel X210CR12 (Č4150) with hardness 58 HRC by orthogonal arrays with five levels (coded by:1,2,3,4 and 5), Table 1.

Table 1. Machining parameters and their levels

Symbol	Parameters	Levels				
		1	2	3	4	5
A	Spindle speed, $n$ ( $\text{min}^{-1}$ )	3981	4777	5573	6369	7169
B	Feed per tooth, $f_z$ (mm/tooth)	0,018	0,024	0,030	0,036	0,042
C	Axial depth of cut, $a_p$ (mm)	0,04	0,08	0,12	0,16	0,20
D	Radial depth of cut, $a_e$ (mm)	0,20	0,40	0,60	0,80	1,00

During the experiments, orthogonal cutting forces were measured using Kistler dynamometer and sampled using a PC based data acquisition system with LabVIEW software. The resultant cutting force was calculated using measured value of orthogonal cutting forces  $F_x$ ,  $F_y$  and  $F_z$  [5]:

$$F_R = \sqrt{F_x^2 + F_y^2 + F_z^2} \quad (1)$$

Roughness measurement has been done using mobile roughness measurement device "MarSurf PS1". Material removal rate was calculated by equation (2) [5]. Equation (2) permits to calculate the theoretical material removal rate  $Q$  given a certain radial depth of cut  $a_e$ , a certain axial depth of cut  $a_p$ , a certain cutting tool radius  $r$ , a certain feed per tooth  $f_z$ , spindle speed  $n$  and number of teeth  $z$ :

$$Q = f_z \cdot z \cdot n \cdot \left[ a_p \cdot a_e - a_e \cdot r + \frac{r^2 \cdot \pi}{180} \arcsin\left(\frac{a_e}{2r}\right) + \frac{1}{2} \cdot a_e^2 \cdot \sqrt{r^2 - \left(\frac{a_e}{2}\right)^2} \right] \quad (2)$$

#### 4. RESULTS AND DISCUSSION

Experimental results, together with normalized values of response, values of grey relational coefficient and values of grey relational grade are given in Table 2.

Table 2. Grey relational coefficient, grey relational grade and corresponding S/N ratios

N <sup>o</sup>	Parameters				Measured data of responses			Normalized values of responses x <sub>i</sub> (k)			Grey relational coefficient $\xi_i$			Grey relational grade $\gamma_i$		
	A	B	C	D	F <sub>R</sub> (N)	R <sub>a</sub> (μm)	Q (mm <sup>3</sup> /min)	F <sub>R</sub> S-T-B	R <sub>a</sub> S-T-B	Q L-T-B	F <sub>R</sub>	R <sub>a</sub>	Q	Grade	Rank	S/N ratio L-T-B
1	1	1	1	1	28.66	0.303	1.13	1.00	0.98	0.00	1.00	0.97	0.23	0.69	1	-3.21
2	1	2	2	2	50.72	0.454	5.94	0.74	0.95	0.08	0.39	0.92	0.24	0.53	6	-5.49
3	1	3	3	3	67.29	1.587	16.48	0.55	0.73	0.25	0.26	0.67	0.28	0.41	17	-7.78
4	1	4	4	4	81.76	3.375	34.64	0.38	0.37	0.55	0.21	0.46	0.40	0.34	20	-9.41
5	1	5	5	5	103.52	5.041	62.22	0.12	0.04	1.00	0.16	0.36	1.00	0.55	7	-5.24
6	2	1	2	3	61.21	1.402	7.74	0.62	0.77	0.11	0.30	0.70	0.25	0.43	15	-7.40
7	2	2	3	4	77.61	3.235	20.34	0.42	0.40	0.32	0.22	0.47	0.30	0.33	22	-9.65
8	2	3	4	5	97.05	5.259	41.86	0.20	0.00	0.67	0.17	0.35	0.47	0.30	23	-10.46
9	2	4	5	1	98.34	0.322	13.72	0.18	0.98	0.21	0.17	0.97	0.27	0.46	11	-6.76
10	2	5	1	2	46.31	0.501	6.06	0.79	0.95	0.08	0.44	0.91	0.24	0.54	5	-5.28
11	3	1	3	5	91.74	4.719	21.28	0.26	0.11	0.33	0.18	0.38	0.31	0.28	25	-11.10
12	3	2	4	1	86.55	0.362	8.53	0.32	0.97	0.12	0.19	0.95	0.25	0.46	12	-6.66
13	3	3	5	2	105.75	0.523	26.45	0.09	0.94	0.41	0.15	0.90	0.34	0.44	13	-7.05
14	3	4	1	3	41.19	1.328	8.42	0.85	0.78	0.12	0.53	0.71	0.25	0.51	9	-5.89
15	3	5	2	4	76.84	3.640	26.62	0.43	0.32	0.42	0.22	0.44	0.34	0.33	21	-9.72
16	4	1	4	2	82.15	0.514	14.47	0.37	0.94	0.22	0.21	0.90	0.28	0.46	14	-6.76
17	4	2	5	3	94.40	1.602	35.77	0.23	0.73	0.57	0.17	0.66	0.41	0.39	16	-8.14
18	4	3	1	4	29.58	1.697	9.50	0.99	0.71	0.14	0.94	0.65	0.26	0.58	2	-4.74
19	4	4	2	5	74.48	4.851	30.29	0.46	0.08	0.48	0.23	0.37	0.36	0.31	24	-10.14
20	4	5	3	1	73.52	0.463	12.78	0.47	0.95	0.19	0.24	0.92	0.27	0.48	10	-6.44
21	5	1	5	4	93.17	3.405	39.43	0.24	0.37	0.63	0.18	0.46	0.44	0.33	19	-9.52
22	5	2	1	5	43.98	3.252	8.96	0.82	0.40	0.13	0.48	0.47	0.25	0.41	18	-7.66
23	5	3	2	1	48.60	0.225	6.88	0.77	1.00	0.09	0.41	1.00	0.25	0.57	3	-4.95
24	5	4	3	2	81.67	0.309	24.31	0.38	0.98	0.38	0.21	0.97	0.32	0.49	8	-6.19
25	5	5	4	3	113.74	1.050	55.97	0.00	0.84	0.90	0.14	0.77	0.74	0.51	4	-5.78

The mean of the grey relational grade for each level of the parameter is summarized and shown in Table 3. In addition, the total mean of the grey relational grade for the 25 experiments is also calculated and listed in Table 3. Figure 1 shows the grey relational grade graph for the levels of the processing parameters. Basically, the larger the grey relational grade, the better is the multiple performance characteristics. The results of ANOVA for the grey grade values are represented in Table 4. The results of the ANOVA indicate that the percentage contribution of spindle speed  $n$ , feed per tooth  $f_z$ , axial depth of cut  $a_p$  and radial depth of cut  $a_e$  influencing the multiple performance characteristics were 12.65%, 4.93%, 27.18% and 37.99% respectively. Radial depth of cut was found to be the major factor affecting the resultant cutting force, the surface roughness and the material

Table 3. Response table for the mean grey relational grade

N <sup>o</sup>	Factors		Grey relational grade $\gamma$					Max-Min (Delta)	Rank
			Level						
			1	2	3	4	5		
1	Spindle speed $n$ (min <sup>-1</sup> )	A	0.50*	0.41	0.40	0.44	0.46	0.10	3
2	Feed per tooth $f_z$ (mm/z)	B	0.44	0.43	0.46	0.42	0.48*	0.06	4
3	Axial depth of cut $a_p$ (mm)	C	0.55*	0.43	0.40	0.42	0.44	0.15	2
4	Radial depth of cut $a_e$ (mm)	D	0.53*	0.49	0.45	0.38	0.37	0.16	1

Total mean value of the grey relational grade = 0.45; \*optimal level

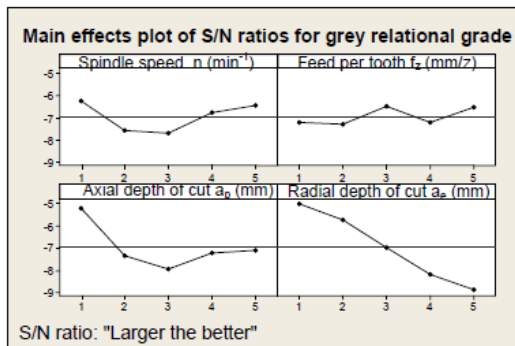


Figure 1. Mean plot for the grey relational grade

## 5. CONCLUSION

This study has concentrated on the application of Taguchi method coupled with Grey relation analysis for solving multi criteria optimization problem in the field of ball-end milling process. Effectiveness of this method was verified by test experiment. The response characteristics of the ball-end milling operations, such as the resultant cutting force, the surface roughness and the material removal rate are greatly enhanced by using this method.

## 6. REFERENCES

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removal rate. After evaluating the optimal parameter settings, the next step is to predict and verify the enhancement of quality characteristics using the optimal parametric combination. Table 5 indicates the comparison of the predicted resultant cutting force, surface roughness and material removal rate with that of actual by using the optimal ball-end milling conditions A1B5C1D1. Good agreement between the actual and predicted results was obtained. Also, improvement in overall Grey relational grade was found to be as 0.25.

Table 4. Results of ANOVA

Factor	Degree of freedom	Sum of squares	Variance	F-ratio	Percent contribution
	DF	S	V	F	P(%)
A	4	0.032	0.013	1.47	12.65
B	4	0.013	0.008	0.57	4.93
C	4	0.070	0.003	3.15	27.18
D	4	0.097	0.017	4.40	37.99
Other errors	8	0.044	0.024		17.25
Total	24	0.257			100

Table 5. Results of confirmation test

	Initial factor settings	Optimal process conditions	
		Prediction	Experiment
Factor levels	A1B3C3D3	A1B5C1D1	A1B5C1D1
$F_R$ (N)	67.29	-	42.42
$R_a$ ( $\mu$ m)	1.587	-	0.278
$Q$ (mm <sup>3</sup> /min)	16.48	-	2.64
S/N ratio	-7.78	-1.91	-3.58
Grey relational grade	<b>0.41</b>	<b>0.73</b>	<b>0.66</b>