

## **DAMAGE IDENTIFICATION IN BEAMS USING ADOPTED FUNCTIONAL MINIMIZATION**

**Elma Ekinović, Nedim Hodžić, Sabahudin Ekinović**  
**University of Zenica**  
**Bosnia and Herzegovina**

### **ABSTRACT**

*A technique for damage identification using bending frequencies of beams is presented in the paper. The numerical values of natural frequencies are calculated using the beam FEA model and then regression relations were established between the first four natural frequencies and damage parameters (location and depth). Based on these regression relations, the estimation of damage location and depth were estimated by experimentally obtained frequency values through minimizing the functional using the software MATHEMATICA 6.0. The proposed damage identification technique was used to estimate damage parameters in 28 damage scenarios (7 locations and 4 damage depths). The results of identification are good and promising.*

**Key words:** beams, bending frequencies, damage location, damage depth

### **1. INTRODUCTION**

Modal parameters (natural frequencies, mode shapes and modal damping) of a structure are directly related to its physical characteristics (mass, stiffness and damping characteristics). Therefore, damage in the structure should be detected and identified by measuring the changes of the structure vibration characteristics. This is the base for vibration based non-destructive structural health monitoring which was investigated during the past few decades. The idea of using modal parameters for damage identification appeared in 1940s and great efforts are dedicated to develop more efficient identification techniques. Different methods that use changes in modal parameters can be found in literature, for instance in [1, 2, 3].

Here presented damage identification technique uses changes in natural frequencies and this paper is aimed to show and estimate the efficacy of the proposed technique.

### **2. AN OVERVIEW OF DAMAGE IDENTIFICATION TECHNIQUE**

Modal parameters (natural frequencies, mode shapes and modal damping) of a structure are directly related to its physical characteristics (mass, stiffness and damping characteristics). Therefore, damage in the structure should be detected and identified by measuring the changes of the structure's vibration characteristics. For the proposed identification technique, several steps are necessary to perform. Firstly, the numerical model of the real beam structure should be established in some of the available software packages, mostly based on finite elements. Then, a numerical analysis of undamaged and different scenarios of damaged beam model should be performed to obtain several bending frequency values. To relate frequencies and damage parameters (location and depth), a nonlinear regression analysis in some statistical software should be done to obtain the necessary regression relations. At the beginning, the experimental measurement of bending frequencies for the real intact beam structure should be done. From time to time measurements should be repeated to obtain current values of frequencies. If in the meantime a single damage occurred, new values of frequencies together with those previously obtained should be put in the adopted functional and the damage parameters would be obtained as those giving the minimum value to the adopted functional.

### **3. DAMAGE IDENTIFICATION TECHNIQUE IN DETAILS**

#### **3.1. Numerical modal analysis of a beam structure**

A simple case of a free-free beam was modeled in software I-DEAS Master Modeler 9. The beam characteristics were: beam length  $L_B=400$  mm, height  $H=8.16$  mm, width  $B=8.12$  mm, modulus of elasticity  $E=2.068 \times 10^{11}$  Pa, mass density  $\rho=7820$  kg/m<sup>3</sup> and Poisson's coefficient  $\nu=0.29$ .

The damage was shaped as an open notch of 1 mm width perpendicular to the beam axis, Figure 1. The location of the damage  $L_D$  and its depth  $d$  were varied and natural bending frequencies  $f_i^{NU}$  and  $f_i^{ND}$ ,  $i=1,2,3,4$ , was numerically obtained. Here, the index NU means numerical value for undamaged beam and ND for damaged beam. More on this topic can be found in [4].

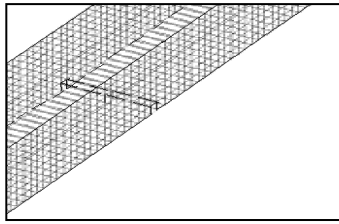


Figure 1. FEM model of damaged beam

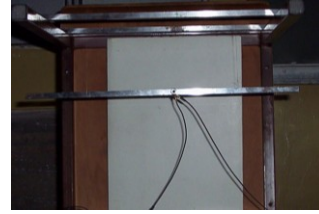


Figure 2. The beam sample in free-free state

### 3.2. Experimental modal analysis of beam samples

To estimate the strength of the proposed identification technique, experimental modal analysis of seven beam samples were performed. The instruments used were: PC, frequency analyzer HP 3567A, interface HP82335A, accelerometers B&K 4394 and impact hammer B&K 8202 with load cell B&K 8200.

To attain a free-free state, each beam sample was hung by two silicon ropes, Figure 2. The damage was made by a cut with a 1mm thin blade at one of the seven chosen locations. Measurements were done firstly on undamaged beam and then on the damaged beam with a damage of 1 mm depth. Then, measurements were repeated for three other nominal depths.

The values of frequencies obtained numerically and experimentally differ less or more due to modeling and measurement errors that are inevitable in reality. In some way this enabled to estimate the robustness of the proposed technique. In reality, however, the efforts should be made to establish the numerical model that is the best representation of the real beam, so the better identification results should be obtained. Also, it was impossible to cut accurately the nominal depth of the notches using the ordinary saw cut. More on the topic of experimental measurements can be found in [5].

### 3.3. Establishment of nonlinear regression relations for natural frequencies

The numerical frequencies  $f_i^{NU}$  and  $f_i^{ND}$  are then used as input data to find regression relations with damage parameters  $D = d/H$  and  $L = L_D/L_B$ . Using software STATISTICA 6.0, the best fitting results are obtained assuming the quadratic influence of the relative depth  $D$  and polynomial influence of the relative location  $L$ .

*The regression relations for the beam under consideration are found in the general form:*

$$f_{iR}(D, L) = f_i^{NU} \left[ 1 - a_i D^2 \sum_{j=0}^k b_{ij} L^j \right], \quad \dots (1)$$

where  $f_i^{NU}$ ,  $i=1,2,3,4$ , are numerical frequencies of the undamaged beam, Table 1, and  $k=4,5,6,7$  for  $i=1,2,3,4$  respectively. The  $a_i$  and  $b_{ij}$  are regression coefficients obtained using the Nonlinear Estimation option.

### 3.4. Adopted identification functional

The estimation of damage location and depth was obtained by finding the minimum of the functional  $FUN(D, L)$ , which is defined as:

$$FUN(D, L) = \sum_{i=1}^4 \left[ \frac{\left( \frac{f_i^{NU} f_i^{ED}}{f_i^{EU}} \right)^2 - (f_{iR}(D, L))^2}{\left( \frac{f_i^{NU} f_i^{ED}}{f_i^{EU}} \right)^2} \right]^2, \quad \dots (2)$$

where  $f_i^{EU}$  and  $f_i^{ED}$  are the  $i^{\text{th}}$ -natural frequencies of undamaged and damaged beam obtained experimentally. The proposed functional is similar to that given in [6], which is based on the assumption that the ratio  $f_i^{NU} / f_{iR}(D, L)$  is close to the  $f_i^{EU} / f_i^{ED}$ . The minimum was searched inside the bounds for  $D$  (from 0 to 0.5) and  $L$  (from 0 to 0.5), which were adopted during numerical analysis.

### 3.5. Example of obtaining the location and depth of a damage

Here are the results of damage identification for the beam sample with  $L_D=110$  mm (with  $L_{real}=0.275$ ). The frequencies obtained numerically and experimentally for the undamaged beam sample and the beam sample after damaging at  $L_{real}=0.275$  with four values of damage depth are given in Table 1.

Table 1. Frequencies of the undamaged and damaged beam (damage scenario with  $L_{real}=0.275$ )

Frequencies of the undamaged beam		Experimental frequencies of the beam damaged at $L_{real}=0.275$			
Numerical	Experimental	$D_{real}=0.125$	$D_{real}=0.25$	$D_{real}=0.375$	$D_{real}=0.5$
$f_1^{NU}=264.2195$	$f_1^{EU}=259.875$	$f_1^{ED}=259.375$	$f_1^{ED}=258.875$	$f_1^{ED}=256.75$	$f_1^{ED}=252.875$
$f_2^{NU}=737.647$	$f_2^{EU}=727.75$	$f_2^{ED}=723.75$	$f_2^{ED}=718.38$	$f_2^{ED}=703.5$	$f_2^{ED}=678.75$
$f_3^{NU}=1416.678$	$f_3^{EU}=1398.63$	$f_3^{ED}=1392.75$	$f_3^{ED}=1390.6$	$f_3^{ED}=1374.15$	$f_3^{ED}=1353$
$f_4^{NU}=2354.085$	$f_4^{EU}=2336$	$f_4^{ED}=2326$	$f_4^{ED}=2330$	$f_4^{ED}=2334.5$	$f_4^{ED}=2329$

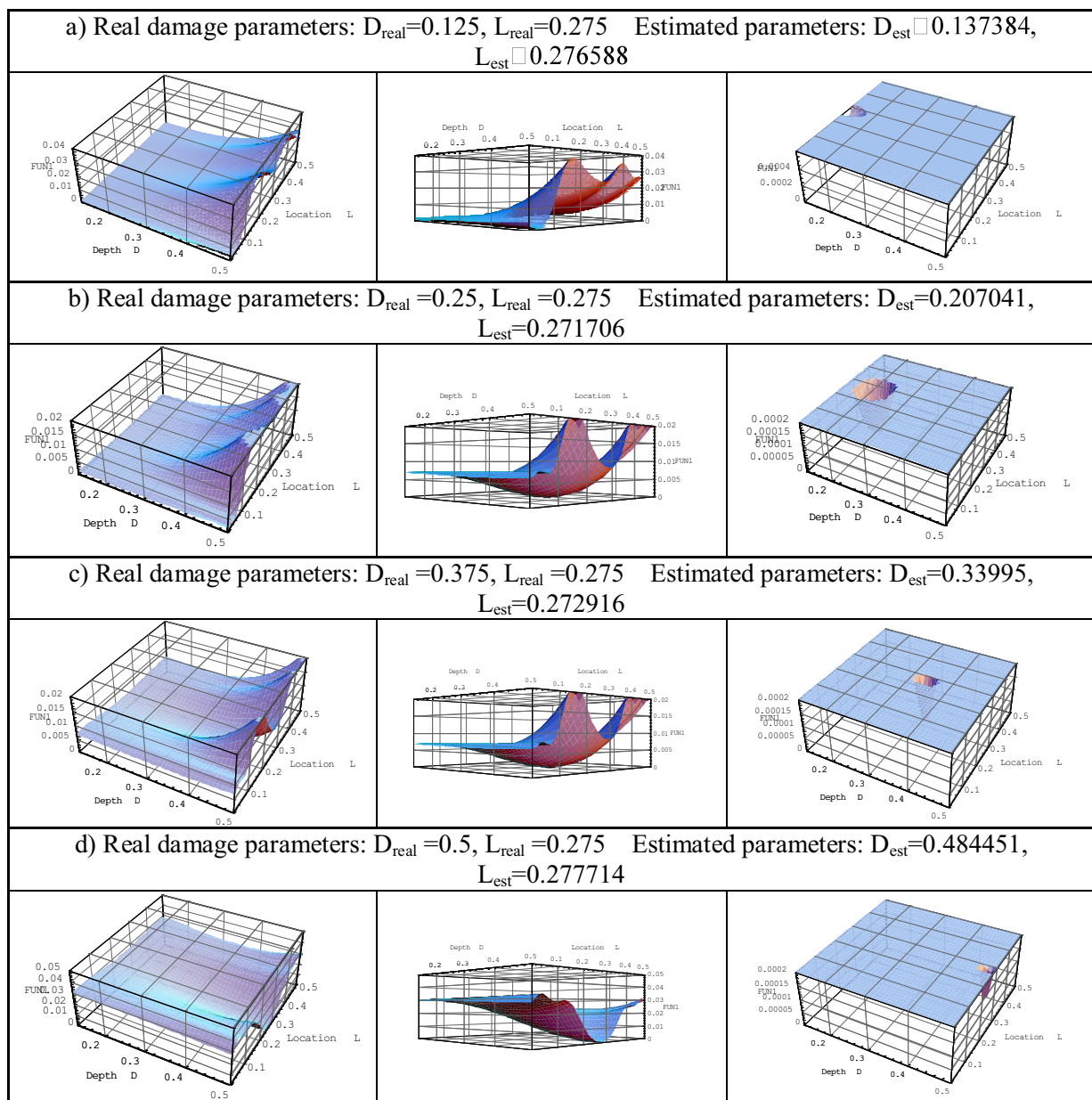


Figure 3. Graphical presentations of functional in two views and the zoomed area of functional minimum around the  $L_{est}$  and  $D_{est}$ , for the numerical case  $L_{real}=0.275$  and four depths of the damage

Using the calculated and measured frequencies, the minimum of the identification functional  $FUN(D,L)$  was found in software Mathematica 5.1. Figure 3 shows the results of damage identification i.e. the numerical values of  $L_{est}$  and  $D_{est}$  obtained by use of FindMin option in Mathematica 5.1 for the beam sample with  $L_{real}=0.275$  and four depths of the damage. There are also graphical presentations of the functional in two views and the zoomed area of the functional minimum around the  $L_{est}$  and  $D_{est}$ .

#### 4. THE RESULTS OF DAMAGE DEPTH IDENTIFICATION

The identification procedure explained in chapter 3 was done for all 28 cases of damaged states (7 locations by 4 depths) and the results of damage identification are shown in Fig. 4.

For four cases that refers to the location of  $L_{real}=0.05$ , the Mathematica software couldn't find the accurate numerical values of  $D_{est}$  and  $L_{est}$  for the functional minimum but using the appropriate zoomed views, the characteristic values of  $D_{est}$  and  $L_{est}$  have been defined approximately.

The identification results on Figure 4 show that, despite all modeling and measuring errors, the results of the used identification technique are quite accurate.

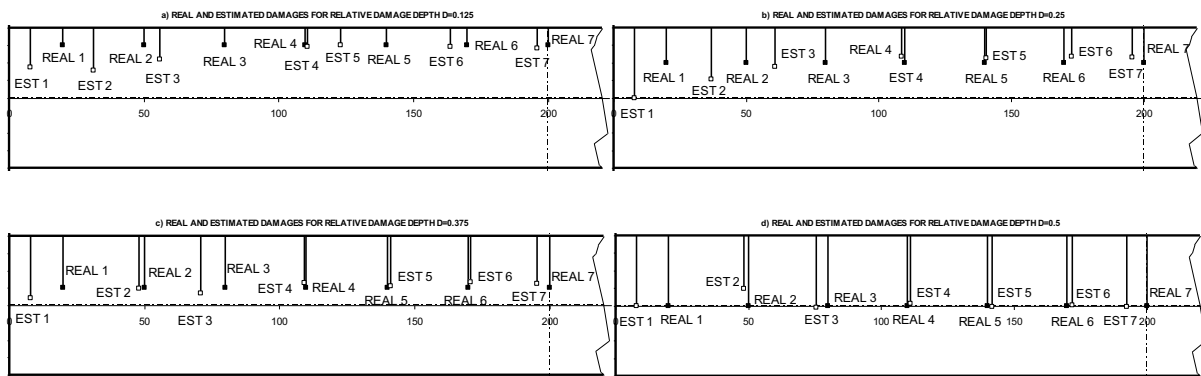


Figure 4. Damage identification results for all 28 damage scenarios

#### 5. CONCLUSIONS

The present paper shows the damage identification technique based on numerical, regression and experimental values of bending frequencies.

Although the accuracy of the technique depends on many factors, such as the quality of numerical model, estimation of regression relationships, quality of frequency measurements, the results of identification are quite satisfactory.

The identification results could be improved using better mesh refinement and higher number of numerical calculations which would provide better base for establishing the regression relations.

It would be interesting to explore if the presented method could be used in beams of different cross-sections or using natural frequencies of other types of vibrating modes.

#### 6. REFERENCES

- [1] Dimarogonas AD. Vibration of Cracked Structures - A State of the Art Review. Engineering Fracture Mechanics , 55/5: pp 831-857, 1996.
- [2] Doebling SW, Farrar CR, Prime MB. A Summary Review of Vibration-Based Damage Identification Methods. Shock and Vibration Digest, 30: pp 91-105, 1998.
- [3] Salawu OS. Detection of Structural Damage through Changes in Frequency: A Review. Engineering Structures, 19: pp 718-723, 1997.
- [4] Ekinović E., Doleček V., Dolinšek S., Hodžić N.: An Approximate Technique for Damage Identification in Beams using Nonlinear Regression Analysis of Bending Frequency Changes, Annals of the Faculty of Engineering Hunedoara - Journal of Engineering, Tome VII, Fascicule 3, ISSN 1584-2673: pp 192-200, 2009
- [5] Ekinović E., Doleček V., Dolinšek S., Hodžić N.: Experimental Verification of an Approximate Technique for Damage Identification in Beams Using Nonlinear Regression Analysis of Bending Frequency Changes, Annals of the Faculty of Engineering Hunedoara - Journal of Engineering, Tome VIII, Fascicule 1, ISSN 1584-2665: pp 43-50, 2010
- [6] Rucevskis. S., Wesolowski, M.: Identification of Damage in a Beam Structure by using Mode Shape Curvature Squares, Shock and Vibration, 17: pp 601-610, 2010