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A SENSITIVITY ANALYSIS OF OPERATING TEMPERATURES TO CHANGES OF SPUR GEAR GEOMETRY

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ABSTRACT
Friction that occurs during operation of the gears, on the meshing flanks of its teeth, causes heating of the teeth, which could lead to the fomation of different surface defects. Intensity of this friction depends on many parameters such as gear geometry, rotational speed, load, lubrication, etc. Proper choice of gear geometry may reduce the friction and operating temperature of the gear, preventing surface defects. Therefore, this paper deals with a sensitivity analysis of operating temperature to changes of gear geometry over a range of applied load and rotational speed. The control volume finite element model – CVFEM, using three-noded triangular elements, has been employed to investigate spur gear frictional heating over a range of gear dimensions, such as module, pressure angle and coefficient of profile shift.
Keywords: *spur gear, operating temperature, gear geometry, CVFEM*

1. INTRODUCTION
Previous studies of the frictional heating of the gears are mainly in the field of numerical modeling and experimental measurements [1-5]. Developed numerical models were used to investigate the influence of gear geometry and other influental parameters on the distribution of operating temperature along the pressure line of meshed gears. The influence of face width, outside diameter and pitch diameter [2,3], as well as, face width and module [4] on the distribution of operating tamperature of the gear were investigated by these models. The result of these investigations are insufficient for complete understanding of the gear geometry influence to distribution of its operational temperature. Therefore, this paper deals with a sensitivity analysis of operating temperature to changes of medium speed spur gear geometry over a range of applied load and rotational speed in order to achieve deeper insight into influence of gear geometry to distribution of its operating temperature. Control volume finite element model [5], using three-noded triangular elements, has been employed to investigate spur gear frictional heating over a range of gear dimensions, such as module, pressure angle and coefficient of profile shift.

2. METHODS
The governing equation of two-dimensional transient heat conduction is described by the following equation:

$$\frac{\partial T}{\partial t} = \alpha \cdot \nabla^2 T \tag{1}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, α is thermal diffusivity and $T=T(x,y)$ is temperature changing with time t and position (x,y). Since each gear tooth during operation passes through the same cycle that is consisted of frictional heat, then convective cooling and conduction, transient heat conduction of the gear can be analysed by a single-tooth model shown in Figure 1.

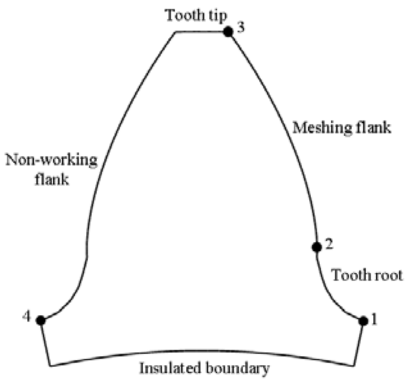

Fig.1 Single tooth model of the gear

Convective boundary conditions for different boundaries are specified as follows:

$$-\left.\frac{\partial T}{\partial n}\right|_{1-2}=-\left.\frac{\partial T}{\partial n}\right|_{3-4}=h\cdot\left(T-T_{oil}\right)$$
$$-\left.\frac{\partial T}{\partial n}\right|_{2-3}=h\cdot\left(T-T_{oil}\right)+q_F \qquad (2)$$
$$\left.\frac{\partial T}{\partial n}\right|_{4-1}=0$$

where h is heat transfer coefficient, q_F is frictional heat flux, T_{oil} is lubricant temperature.

Model for determining the friction coefficient of contact surfaces proposed by Winter and Michaelis [6], which is used for this research, is given by the following expression:

$$\mu=0.002\cdot\left(\frac{2\cdot F_N}{b\cdot\cos\varphi\cdot\left(v_{t1}+v_{t2}\right)\cdot R}\right)^{0.2}\cdot\eta^{-0.05}\cdot X_R \qquad (3)$$

where where F_N is normal force at meshing point, b is gear width, φ is pressure angle, R is equivalent radius of curvature at meshing point, v_{t1} and v_{t2} are velocities in the direction of the tangent at meshing point of the pinion and gear, η is dynamic viscosity, and X_R is roughness factor. Determination of F_N, R, v_{t1} and v_{t2}, η, and X_R is fully explained in [5]. The heat flux generated by friction of the teeth of the pinion and gear in an arbitrary meshing point, according to Long et al. [4] can be determined by the following equations:

$$q_1=\beta\cdot\gamma\cdot\mu\cdot p\cdot\left(v_{t1}-v_{t2}\right) \text{ and } q_2=\left(1-\beta\right)\cdot\gamma\cdot\mu\cdot p\cdot\left(v_{t1}-v_{t2}\right) \qquad (4)$$

Contact pressure p at meshing point is determined by Hertzian contact theory as the average compressive pressure for a meshing point as explained in [5]. The heat conversion factor γ is defined to be 0.95 and the partition constant of the heat between the contact areas of the meshing teeth β is 0.5 [4]. During each revolution every tooth of the pinion and gear is only once at the contact when receives heat input which is equal to the average frictional heat flux, which can be expressed as:

$$q_{F1}=\frac{a\cdot n_1}{60\cdot v_{t1}}\cdot q_1 \text{ and } q_{F2}=\frac{a\cdot n_2}{60\cdot v_{t2}}\cdot q_2 \qquad (5)$$

where n_1 and n_2 are rotational speeds of the pinion and gear, and a is width of the teeth contact area.

Lubrication and cooling of the pinion and gear is realized by jet lubrication. DeWinter and Blok [7] have developed a model to estimate the heat transfer coefficient on the flank of the gear tooth for this lubricating and cooling method:

$$h=\sqrt{\frac{n}{2\cdot\pi}\cdot k\cdot\rho\cdot c}\cdot\left(\frac{v_o\cdot H}{\alpha\cdot r}\right)^{0.25}\cdot q_n \qquad (6)$$

where n is rotational speed, k is lubricant conductivity, ρ is lubricant density, c is lubricant specific heat, v_o is lubricant kinematic viscosity, α is lubricant thermal diffusivity, H is height of meshing point, r is radius of meshing point, and q_n is normalized cooling capacity.

The governing equation of transient heat conduction (1) could be alternatively expressed in the integral form:

$$\frac{d}{dt}\int_A T\cdot dA=\oint_S \alpha\cdot\nabla T\cdot n\cdot dS, \qquad x\in\Omega \qquad (7)$$

where Ω is arbitrary two dimensional domain. In this research domain Ω is gear tooth geometry shown in Figure 2.

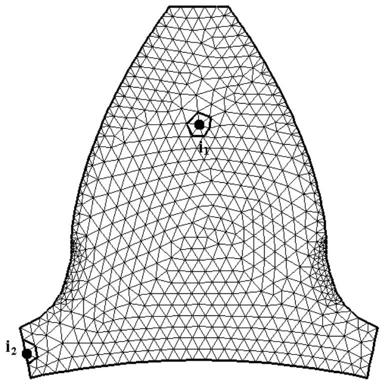

Fig.2 Gear tooth geometry with mesh of triangular elements

The key objective of CVFEM is to reduce the integral form of equation (7) to set of discrete algebraic equations in the unknown nodal values of temperature. First step of this procedure is meshing two dimensional domain into mesh of linear triangular elements shown in figure 2. The continuous unknown field of temperature over triangular elements, can be expressed as the linear combination of the temperature values at nodes placed at the vertices of triangular elements:

$$T(x,y)=\sum_{j=1}^{3} N_j(x,y)\cdot T_j \tag{8}$$

where $N_j(x,y)$ are shape functions and T_j are nodal temperature values. For each node i in the mesh the region of support, shown in figure 2, is identified by counting and listing all the neighboring nodes j that share a common element side with node i. Using numerical integration and shape function aproximations of temperature (8) in each element of i^{th} support, equation (7) is expanded in terms of nodal values of temperature in the region of support. On gethering terms, the resulting equation for node i can be written in the discrete form:

$$a_i \cdot T_i = \sum_{j=1}^{Ns_i} a_{i,j} \cdot T_{i,j} + b_i \tag{9}$$

where a_i is coefficient associated with the unknown nodal values of T_i and $a_{i,j}$ are coefficients associated with the unknown nodal values of $T_{i,j}$ at neighboring nodes in the i^{th} support, and the additional coefficient b_i accounts for the contribution from sources, transients and boundaries. Equation (9) provides an algebraic relationship between the value of temperature at node i and the neighboring nodes j in its region of support. Determination of coefficients a_i, $a_{i,j}$, b_i, B_{Bi} i B_{Ci} is fully explained in [8].

3. RESULTS AND DISCUSSION

Geometric properties of investigated spur gear set are as follows: number of teeth z_1=15 and z_2=16, module m=5.33 mm, pressure angle φ=26°, width b=4.775 mm, and coefficient of profile shift x_1=x_2=0 . The material of the pinion and gear is 665M17 (EN-34) steel with the following mechanical and thermal properties: Young's modulus E=185.42 GPa, Poisson's ratio ν=0.3, thermal conductivity k=41.8 Wm^{-1}K^{-1} and thermal diffusivity α=1.077·10^{-5} m^{2}s^{-1}. Gear set is case-hardened and ground to a surface finish of 0.6 μm Ra. The lubricant oil, applied via spray system, is Mobile Jet II with the following properties: density ρ=998 kgm^{-3}, kinematic viscosity ν_o=27.6·10^{-6} m^{2}s (40 °C), ν_o=5.1·10^{-6} m^{2}s (100 °C), thermal conductivity k=0.1337 Wm^{-1}K^{-1} (37.8 °C), k=0.1278 Wm^{-1}K^{-1} (93.3 °C), specific heat c=2000 Jkg^{-1}K^{-1} (90 °C) and oil temperature T_{Oil}=90 °C.

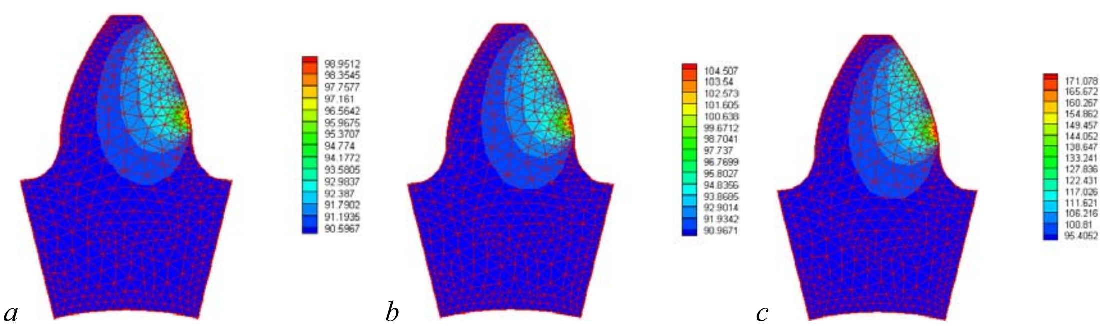

Fig.3 Distribution of tooth temperature of the pinion with m=3 mm for
a) T=17.4 Nm, n=2000 rpm, b) T=17.4 Nm, n=10000 rpm and c) T=73 Nm, n=10000 rpm

In order to investigate an influence of gear geometry on the distribution of its operating temeperature a serie of gear models is generated with variation of module, pressure angle or coefficient of profile shift regarding to gear set used for model verification [5]. Then transient heat analysis of the frictional heating of the pinion was carried out. Some obtained distributions of tooth temperature for different combinations of rotational speeds and load cases are shown in Figure 4. Obtained temperature distributions show that gear temperature increases with increase of rotational speed and even more intense with increase of load. Influence of module, pressure angle and coefficient of profile shift on operating temperature is shown in the following figures with curves of dependencies between gear maximum temperature and mentioned gear dimensions.

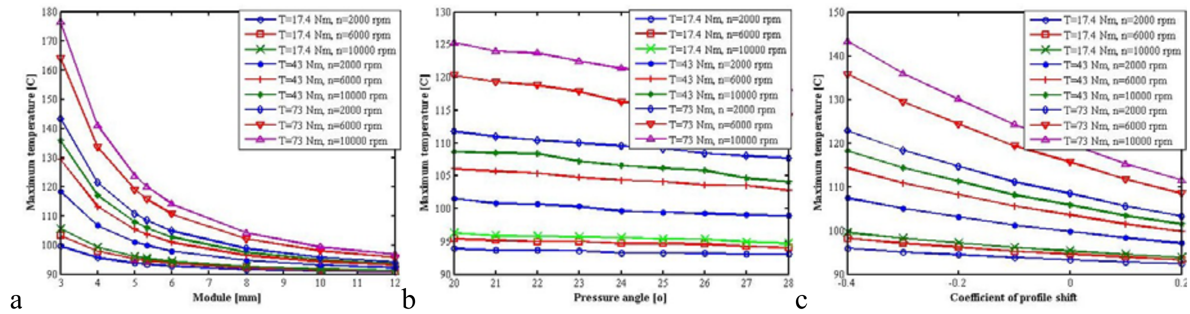

Fig. 5 Temperature dependence on a) module, b) pressure angle and c) coeficient of profile

4. CONCLUSION

The CVFEM models with different combinations of operating conditions and gear geometries are established for evaluating the influences of operating conditions and gear geometry on tooth temperature variations. There are two temperature peaks along meshing flank one below and another above the pitch circle. The maximum gear temperature for each investigated case is at the peak below the pitch circle. Regarding operating conditions, load is more influencing on the gear maximum temperature than rotational speed. High loads also cause more uneven distribution of gear tooth temperature with two very distinct temperature peaks along its meshing flank. Regarding gear dimensions, distribution of the gear tooth temperature is very sensitive to changes of its module, quite sensitive to changes of its coefficient of profile shift and quite insensitive to changes of its pressure angle. Sensitivity of distribution of tooth temperature of the gear to changes of its geometry increases with increase of load magnitude.

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