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OPTIMIZATION OF LONGITUDINALLY STIFFENED PLATES UNDER COMPRESSION

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ABSTRACT

Plate members are widely used in modern lightweight construction. In order to reduce the mass of these structures, while preserving an adequate stiffness at the same time, the thickness of the plate should be as small as possible and the stability is gained by introducing stiffeners. Depending on their rigidity, there are two possible buckling modes for the plate. For flexible stiffeners, the resulting mode is a global one in which the stiffeners bend with the plate. Beyond a particular value of their stiffness, the critical stress of the plate remains constant and the resulting mode is a local buckling mode of the sub-panels of the plate. The goal is to find the cross-section dimensions of stiffeners beyond which the increase in stiffness will no longer affect the value of the critical buckling stress. This paper presents a procedure to get optimized cross-section dimensions for longitudinally stiffened plates under compression with two flat stiffeners. The cross-section is optimized using MATLA algorithms according to the buckling plate theory. The finite element models are used to confirm the optimization process.

Keywords: stiffened plates, buckling, optimization

1. INTRODUCTION

Plate structures are extensively used in engineering applications, such as mechanical and civil engineering, aerospace and naval architecture. They are typically thin structures, where it is important to achieve high stiffness and strength to weight ratio. These structures are usually failed by buckling. The stability of them can be improved by increasing their thickness, but such design is not economical considering the amount of material used. A more economical solution is obtained by keeping the thickness of the plate as small as possible and increasing the stability by introducing longitudinal and/or transversal stiffeners of different size and shape. Thereby, the stiffeners not only carry a portion of load but they also subdivide the plate into smaller sub-panels and thus substantially increase the critical stress σ_{cr} that corresponds to the first buckling mode at which the plate buckles. Depending on the rigidity of the stiffeners, there are two possible buckling modes for the plate [1]. For flexible stiffeners, the resulting mode is a global one when the stiffeners, which are insufficiently rigid, bend with the plate. Beyond a particular value of their rigidity, the critical stress of the plate remains constant and the resulting mode is a local buckling mode of the sub-panels of the plate. In order to minimize the mass of the stiffened plate, the problem is to identify the cross-section dimensions beyond which the increase in stiffness will no longer affect the value of the critical buckling stress. In this paper, a rectangular plate subject to uniform compression is studied, stiffened by two longitudinal flat stiffeners, which are equally spaced on one side of the plate. The critical plate buckling stress is determined using procedure described by Timoshenko & Gere [2] and to the relevant Eurocode (EN 1993-1-5) [3].

2. STABILITY OF LONGITUDINALLY STIFFENED RECTANGULAR PLATE

In what follows, the terms of determining the critical stress according to [2] are presented. In the case of simply supported unstiffened rectangular plate (axbxt), uniformly compressed in one direction (Figure 1), the critical stress is proportional to $(t/b)^2$.

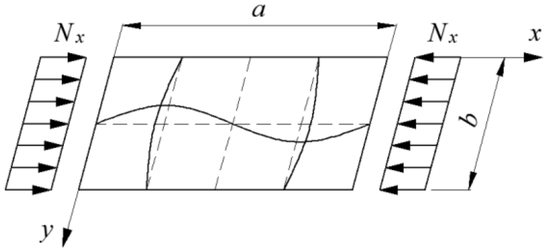

Figure 1. Buckling of simply supported, uniformly compressed rectangular plate

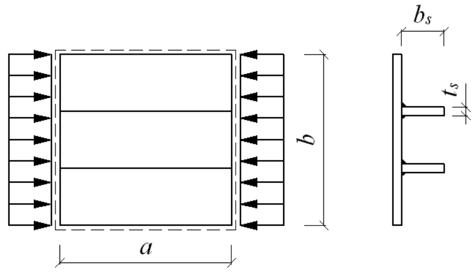

Figure 2. The layout of the stiffened plate

Assuming a case of two longitudinal stiffeners dividing the width of the plate into three parts, at the distances c_i (i=1,2) from the edge y=0, the critical stress is:

$$\sigma_{cr} = \frac{\pi^2 D}{b^2 t} \frac{\left(m^2+\beta^2\right)^2 + 2m^4\left[\gamma_1 \sin^2\left(\frac{\pi c_1}{b}\right) + \gamma_2 \sin^2\left(\frac{\pi c_2}{b}\right)\right]}{\beta^2 m^2 \left\{1+2\left[\delta_1 \sin^2\left(\frac{\pi c_1}{b}\right) + \delta_2 \sin^2\left(\frac{\pi c_2}{b}\right)\right]\right\}} \quad \text{... (1)}$$

where

$$\beta = \frac{a}{b},\ \gamma_i = \frac{EI_i}{bD},\ \delta_i = \frac{A_i}{bt},\ D = \frac{Et^3}{12\left(1-\nu^2\right)} \quad \text{... (2)}$$

I_i and A_i are the second moment of inertia and the area of the stiffener cross section respectively, D is the flexural rigidity of the plate and m is the number of half-sine waves in the longitudinal direction into which the plate buckles. Cross sections of stiffeners may be open or closed, and single or double-sided. In this paper, the optimization of two flat, single-sided stiffeners has been analyzed (Figure 2). In calculating the quantity EI_i, it should take into account that the stiffener is welded to a plate of great width, which results in a considerable increase in its flexural rigidity. The centroid of the cross section, consisting of the stiffener and the plate, will be very near to the axis of their connection, so the moment of inertia of the cross section of the stiffener with respect to this axis, coinciding with the surface of the plate, should be taken in calculating I_i. In this case, that is:

$$I_i = \frac{b_s^3 t_s}{12} + A_i\left(\frac{b_s}{2}\right)^2,\ \ A_i = b_s \cdot t_s,\ \ i=1,2 \quad \text{... (3)}$$

Design rules for stiffened plates assume that torsional buckling of stiffeners is completely prevented when loaded axially. Generally, the following requirement should be fulfilled [3]:

$$\sigma_{cr,T} \geq \theta f_y \quad \text{... (4)}$$

where $\sigma_{cr,T}$ is the elastic critical stress of a stiffener at torsional buckling, f_y is the yield strength, and θ depends on the type of stiffener. For flat stiffeners, the requirement (4) may be rewritten as:

$$\frac{b_s}{t_s} \leq \sqrt{\frac{E}{5.3 f_y}} \quad \text{... (5)}$$

3. FORMULATION OF THE OPTIMIZATION PROBLEM

The optimization problem can be formulated mathematically as follows [4]:

$$\text{Minimize:} \quad f(\mathbf{x}), \quad x_i \in R^n \quad i = 1,\ldots,n$$

subject to:

$$g_i(\mathbf{x}) \le 0, \quad i = 1,\ldots,m \,; \; h_j(\mathbf{x}) = 0, \quad j = 1,\ldots,p \,; \qquad \ldots (6)$$
$$x_k^l \le x_k \le x_k^u, \quad k = 1,\ldots,n$$

where $f(\mathbf{x})$ is the objective function to be minimized. In this study the objective function is the cross-sectional area of one stiffener (both of them have the same dimensions):

$$f(\mathbf{x}) = b_s t_s \qquad \ldots (7)$$

$\mathbf{x}$ is the vector of design variables, $\mathbf{x} = [t_s, b_s]^T$; n=2 is the total number of design variables; x_k^l and x_k^u are the lower and upper bounds of the kth design variable x_k, respectively; $g_i(\mathbf{x})$ is the ith inequality constraint and $h_j(\mathbf{x})$ is the jth equality constraint. The solution process involves selecting a most suitable optimization technique or algorithm to find an optimal solution. When either the objective or any constraint function is nonlinear in terms of the variables, it is generally referred as constrained nonlinear optimization or a nonlinear programming (NLP) problem. In MATLAB, this kind of problems can be solved using the function *fmincon*, which attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate. It uses derivative-based search methods, also known as gradient-based search methods. In such approaches, we estimate an initial design and improve it iteratively, until optimality conditions are satisfied. The general syntax of this function is [5]:

x = fmincon(fun,x0,A,b,Aeq,beq,lb,ub,nonlcon,options) …(8)

where *fun* is the function to be minimized, and *nonlcon* is the function that computes the nonlinear inequality and equality constraints. Both of these functions have been written as separate m-files, where the constrained functions were defined based on the plate buckling theory and to the design rules imposed by EN 1993-1-5.

4. APPLICATION EXAMPLE

Numerical example: The optimal dimensions of two equally spaced single-sided flat stiffeners for the steel plate with dimensions axbxt=1500x1500x7 mm have been determined. The following results were obtained: *Critical stress of unstiffened plate: 16.53 (MPa), Number of half-sine waves: 1, Minimal rib dimensions causing local buckling mode:* t_sxb_s*=8x62.00 mm, Critical stress of stiffened plate: 161 MPa, Surface area of a rib: 496.00 mm*^{2}*, Mass of 2 ribs: 11.61 kg*

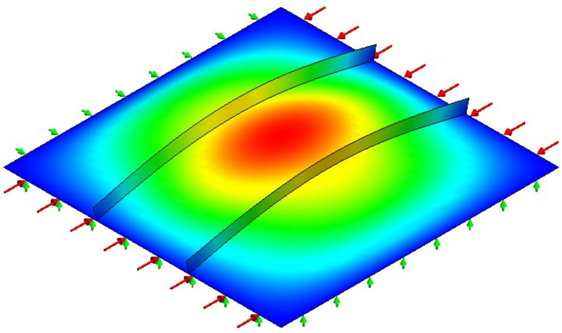

Figure 3. Global buckling mode of the steel stiffened plate (axbxt*=1500x1500x7 mm,* t_sxb_s *= 8x60 mm)*

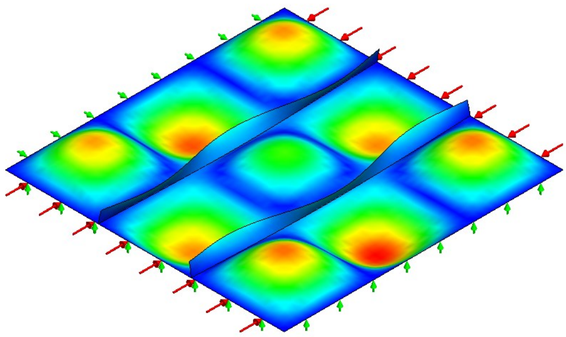

Figure 4. Local buckling mode of the steel stiffened plate (axbxt*=1500x1500x7 mm,* t_sxb_s *= 8x62 mm)*

Since there are only two design variables, the graphical representation of the optimization has been creating through MATLAB code, as shown on Figure 5. The iso-contours of the objective function, i.e. the area of cross section of one stiffener A_i, are plotted in a two-dimensional design space where the axes represent the two design variables, width (t_s) and height (b_s) of the single-sided flat stiffener. The optimized solution, beyond which the increase in stiffness will no longer affect the value of the critical buckling stress, is given by point A_{opt}.

In order to confirm the optimized values, the evaluation design study has been performed in SolidWorks Simulation, which employs a generative method for support of design optimization. The plate model was parameterized and meshed using shell elements. The software runs the study using various combinations of the stiffener cross-section values and it creates different design scenarios. The thickness of the stiffeners determined through MATLAB optimization process was used as an invariable value and the effect of the stiffener height was considered to reach the critical value of the stress at which the plate moves from global to local buckling mode. Finite element analyses are carried out for all scenarios generated. Based on the scenarios evaluated, graphical representation of critical stress on flat stiffener height has been generated (Figure 6). These results confirmed the values gained through MATLAB optimization.

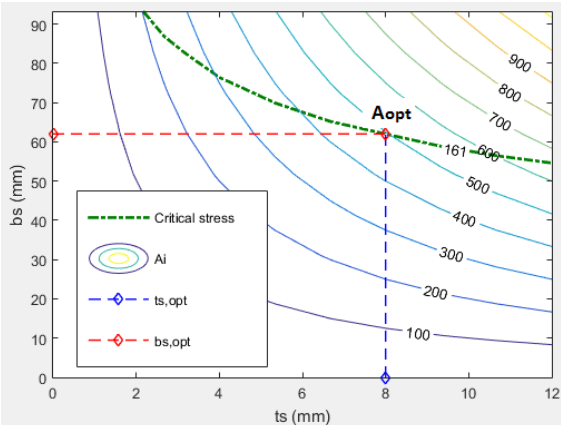

Figure 5. Graphical representation of the MATLAB optimization solution

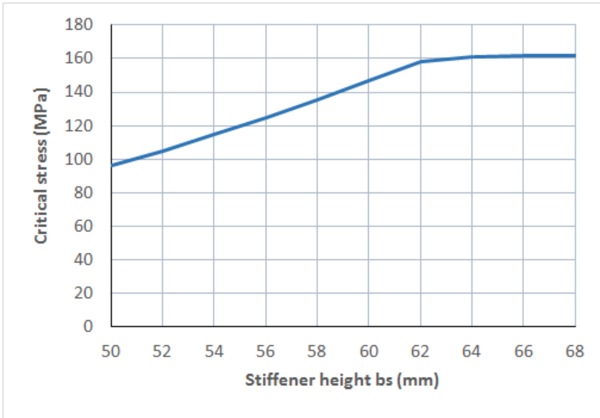

Figure 6. Graphical representation of the influence of flat stiffener height b_s on the critical stress ($axbxt$=1500x1500x7 mm, t_s = 8 mm)

5. CONCLUSION

In this paper, a procedure for optimization of the cross-section dimensions for two longitudinally flat stiffeners for the plate under uniform compression has been presented. The optimization has been done using MATLAB optimization algorithm. The limits of the plate slenderness value and the geometric proportions of the cross-section imposed by the relevant Eurocode are taken into account as constraints on the optimization process. Based on the obtained values of the thickness and the height of the flat stiffeners, FE analysis of the plate buckling was done in SolidWorks Simulation. This analysis confirmed the validity of the algorithm developed in Matlab for determining the values of the stiffener cross-section dimensions beyond which the increase in stiffness will no longer affect the value of the critical buckling stress.

6. REFERENCES

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