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NOVEL MULTIPLIERLESS COMB DECIMATION FILTER BASED ON COMB ZERO-ROTATION

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ABSTRACT
Decimation is process of decreasing sampling rate by an integer, called decimation factor. This process introduces aliasing which may deteriorate the decimated signal and must be eliminated by a filter, called decimation filter. The most simple decimation filter is a comb filter, which has all coefficients equal to unity, and consequently does not require multipliers for its implementation. However, comb filter does not provide enough aliasing attenuation. This paper presents novel comb-based decimation filter with an improved alias rejection in comparison with the original comb filter. This is achieved by rotation of comb zeros from their original position, using simple multiplierless filter. Mathematical background for the design of the filter is provided and explained with one example. The proposed method is illustrated with one example and compared with the original comb filter.

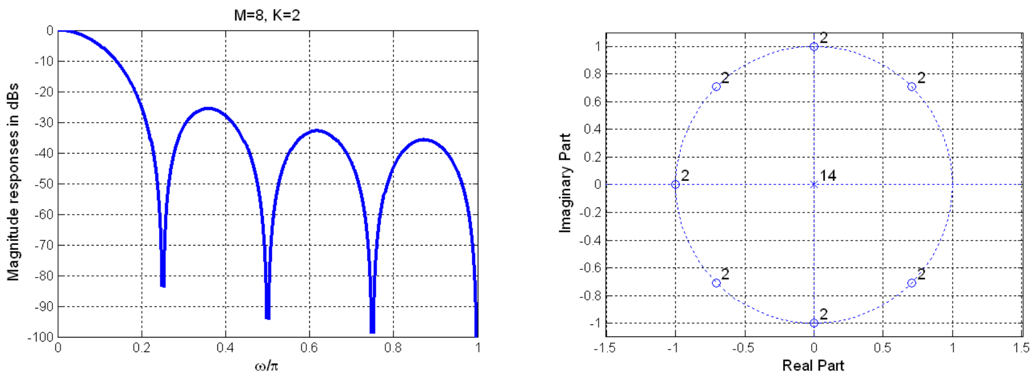
Keywords: decimation, aliasing, comb filter.

1. INTRODUCTION
Decimation is process of decreasing sampling rate by an integer, called decimation factor. Decimation has applications in communications, audio signal processing and Oversampled Sigma Delta Analog/Digital (SD A/D) converters, among others [1]. This process introduces aliasing which may deteriorate the decimated signal and must be eliminated by a filter, called decimation filter. The most simple decimation filter is a comb filter, which has all coefficients equal to unity, and consequently does not require multipliers for its implementation. Its transfer function is given as:

$$H(z)=\left[\frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\right]^{K}=\left[\frac{1}{M}\sum_{k=1}^{M-1}z^{-k}\right]^{K}, \tag{1}$$

where M is the decimation factor and K is the number of the cascaded combs, also called the order of the comb.

Comb filter must have high attenuations in bands around comb zeros, called folding bands. However, comb filter does not provide sufficient attenuations of aliasing in folding bands, as shown in Fig.1 for M= 8, and K=2. The pole-zero-plot in z-plane is also shown. Note that all zeros are doubled in each folding band position, resulting in narrow folding bands. The goal is to increase the width of folding bands and thus increase alias rejection capability. In [2] Presti proposed comb-zero rotation which introduces additional zeros into comb folding bands and thus increases comb alias rejection. However, this method requires two multipliers. Knowing that the comb filter is a multiplierless filter, the goal is



a. Magnitude response. *b. Pole-zero plot.*

Fig.1. Comb filter: M=8, K=2.

to improve comb alias rejection using simple multiplierless filters [3-7].

In this paper we present one new approach to improve comb alias rejection by introducing comb zero-rotation without multipliers. The mathematical background is presented in Section 2 and explained with one example. Next section describes the proposed method and is illustrated with one example.

2. MULTIPLIERLESS ZERO ROTATION

Consider the comb filter with the parameter M which can be expressed as the product of two integers M_1 and M_2. Next we form comb filter $y_1(n)$ of length

$$N_1 = M - M_1 + a\,, \tag{2}$$

where a is an integer, $a < M_1$.

$$Y_1(z) = \frac{1}{N_1}\sum_{k=0}^{N_1-1} z^{-k}\,. \tag{3}$$

In the following we form comb filter $y_2(n)$ of length M_2, expanded by M_1:

$$Y_2(z^{M_1}) = \frac{1}{M_2}\sum_{k=0}^{M_2-1} z^{-M_1 k}\,. \tag{4}$$

The cascade of the filters (3) and (4) is denoted as $Y(z)$:

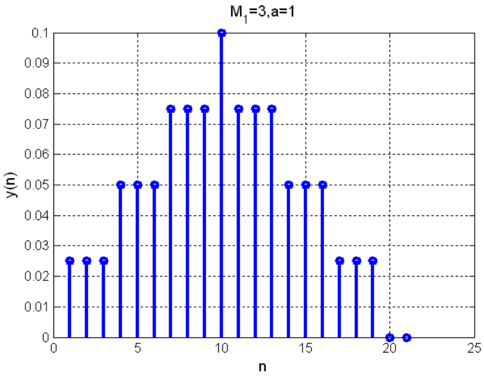
$$Y(z) = Y_1(z)Y_2(z^{M_1})\,, \tag{5}$$

The filter (5) has an impulse response in a stepped triangular form in which the number of samples in each level is equal to M_1, while in the last level is equal to a. The number of levels is equal to M_2. This filter has zeros around the zeros of the original comb filter of length M, and consequently naturally introduces zeros into the original comb folding bands.

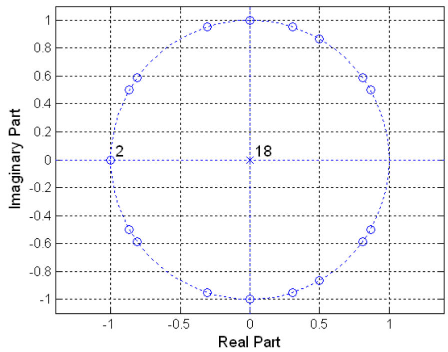
This idea is explained in the following example:

Example 1:

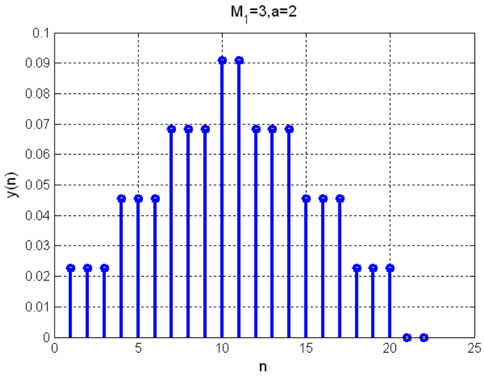
We consider M=12, M_1=3, and M_2=4. The impulse response of the filter (5) has M_2=4 levels, and the number of samples at each level is equal to M_1=3, except at the last level, where it is equal to a. The possible values of a are equal to 1, 2. (The value of a is less than M_1). Figures 1a and 1b show the impulse response of the filter $y(n)$ for a=1 and the corresponding pole-zero plot. Similarly, the impulse response of the filter $y(n)$ for a=2 and the pole-zero plot are shown in Figures 1c and 1d. For the sake of comparison, Figures 1d and 1f show the impulse response of comb filter and the corresponding pole-zero plots. Observe that the filters $y(n)$ for both values of a have the zeros around the comb zeros. Consequently, the cascade of the filter $y(n)$ with the comb filter introduces the additional zeros into the comb folding bands and thus increases the comb alias rejection. This is confirmed in Figure 3 which compares the magnitude responses of the cascade of comb, and filter $y(n)$ for both values of a. Observe that the value a=2 provides slightly better alias rejection than a=1.



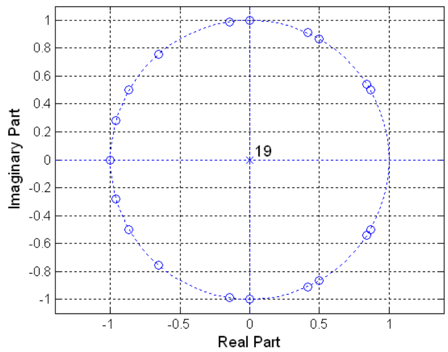
a. *Impulse response.*



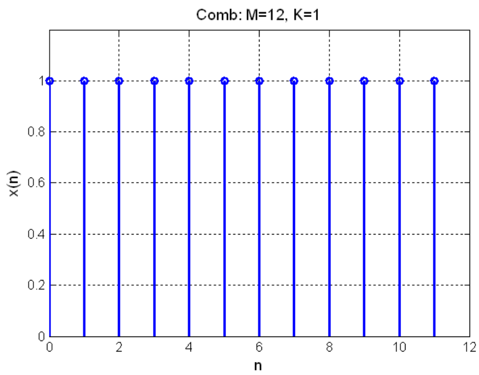
b. *Pole-zero plot.*



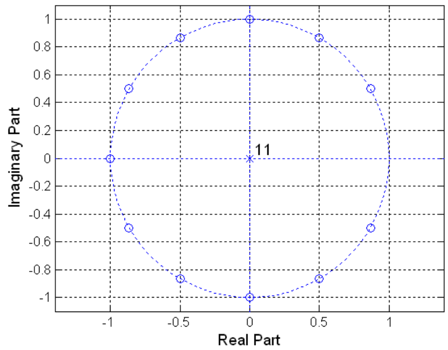
c. *Impulse response.*



d. *Pole-zero plot.*

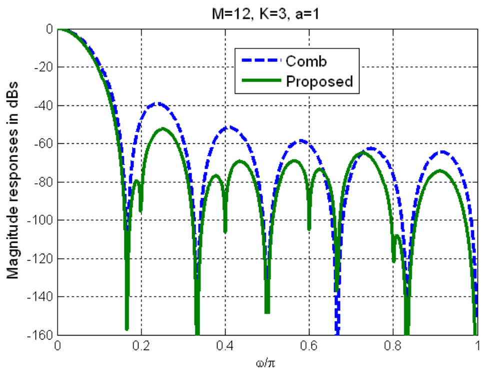


e. *Impulse response.*

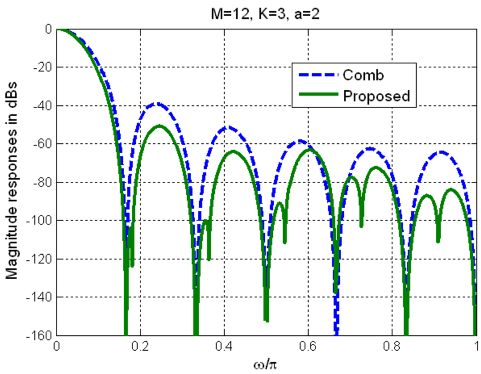


f. *Pole-zero plot.*

Figure 2: Impulse responses and pole-zero plots for filter y(n) for two values of a, and comb filter.



a. *a=1.*



b. *a=2.*

Figure 3: Magnitude responses

3. PROPOSED METHOD

The proposed filter is the cascade of comb filter and the filter Y(z). The transfer function is given as:

$$H_p(z) = H(z)Y(z), \tag{6}$$

where $H(z)$ is the comb transfer function from (1) and the transfer function Y(z) is according to (1) and (5) equal to:

$$Y(z) = Y_1(z)Y_2(z^{M_1}) = \frac{1}{N_1}\frac{1-z^{-N_1}}{1-z^{-1}}\frac{1}{M_2}\frac{1-z^{-M_1M_2}}{1-z^{-M_1}} = \frac{1}{N_1}\frac{1-z^{-N_1}}{1-z^{-1}}\frac{1}{M_2}\frac{1-z^{-M}}{1-z^{-M_1}}, \tag{7}$$

where N_1 is given in (2).
The method is illustrated in the following example.
Example 2:
We consider M=15, M_1=3 and M_2=5, K=4, and a=2. From (2) N_1=14.
The magnitude responses of the proposed filter (6) and the comb filter are contrasted in Figure 4.

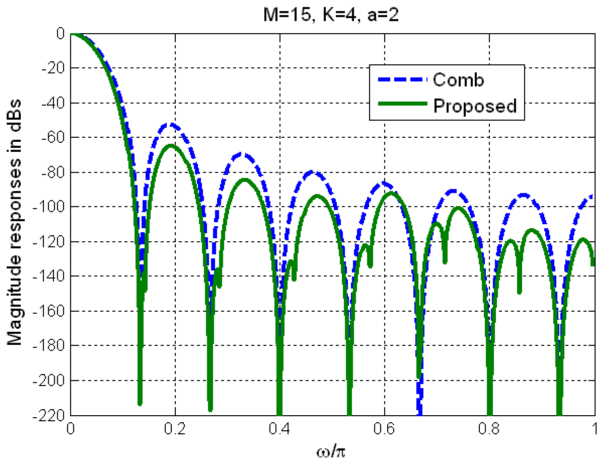


Figure 4: Magnitude responses of the proposed filter and comb filter.

4. CONCLUSION

This paper presents simple method to improve comb alias rejection by introducing additional zeros into comb folding bands provided by a simple multiplierless filter. This filter has stepped triangular impulse response in which the number of levels is equal to M_2, the number of samples at each level is equal to M_1, and the number of samples at last level is equal to a, where M_1 and M_2 are integers, $M= M_1 M_2$, $M_1 < M_2$, and a is the parameter of design, $a<M_1$.

5. REFERENCES

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